Shroff S.R. Rotary Institute of Chemical Technology

Block Diagram reduction techniques

Subject name : Control System Engineering (2150909)

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Introduction

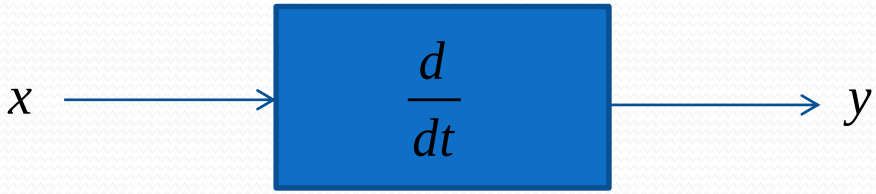
- Block diagram is a shorthand, graphical representation of a physical system, illustrating the functional relationships among its components.

OR

- A Block Diagram is a shorthand pictorial representation of the cause-and-effect relationship of a system.

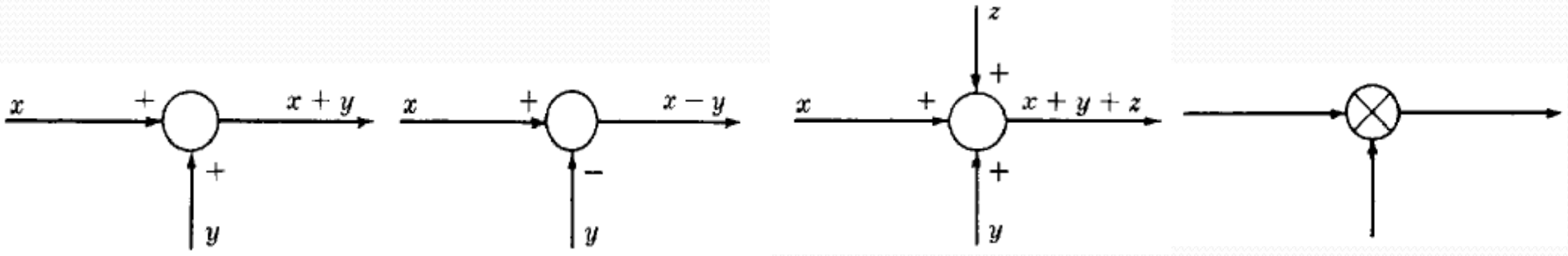
Introduction

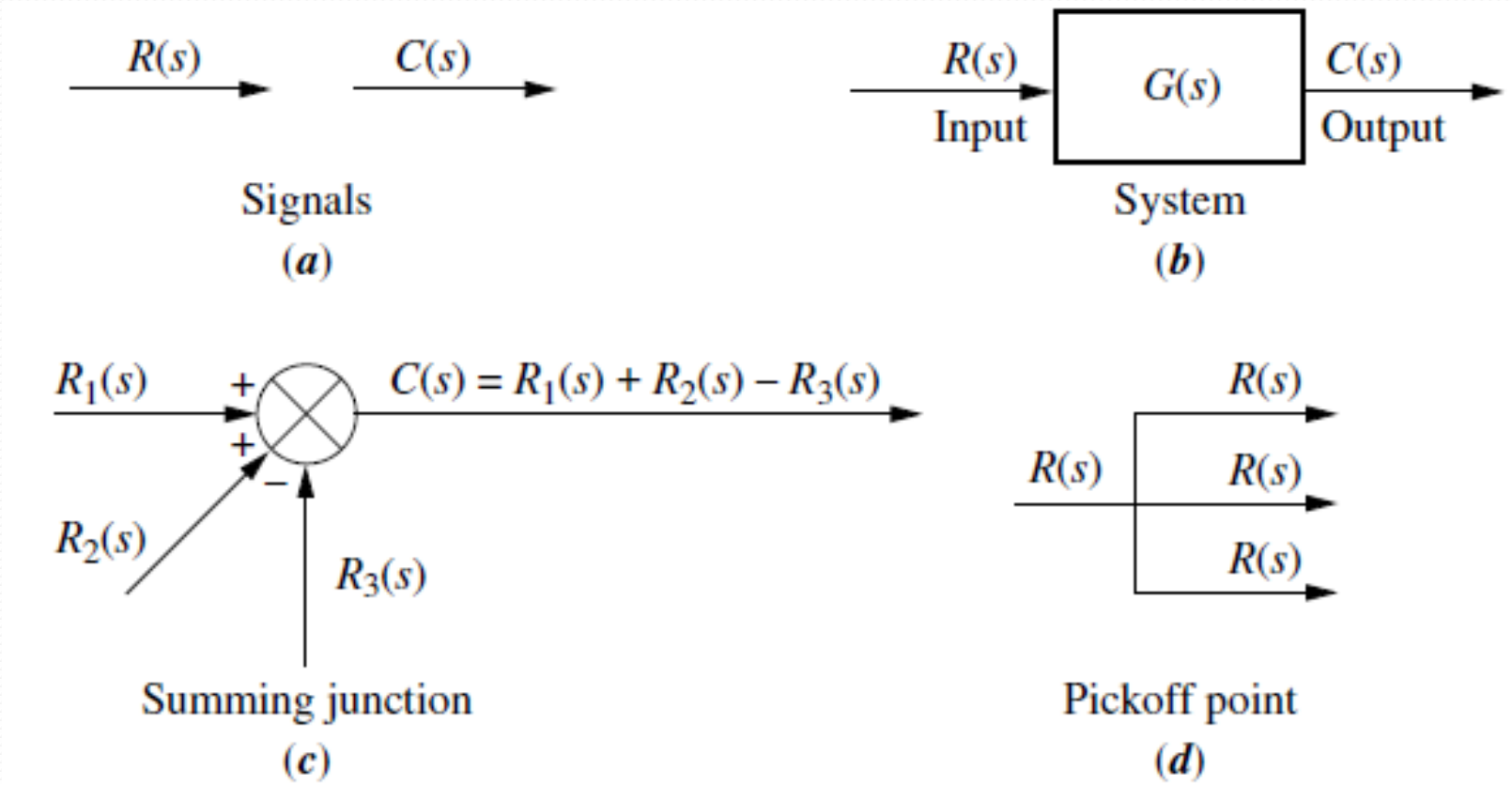
- The simplest form of the block diagram is the single ***block, with one input and one output.***
- The interior of the rectangle representing the block usually contains a description of or the name of the element, or the symbol for the mathematical operation to be performed on the input to yield the output.
- The arrows represent the direction of information or signal flow.



Introduction

- The operations of addition and subtraction have a special representation.
- The block becomes a small circle, called a summing point, with the appropriate plus or minus sign associated with the arrows entering the circle.
- Any number of inputs may enter a summing point.
- The output is the algebraic sum of the inputs.
- Some books put a cross in the circle.




R(s)
C(s)
Signals
(a)
R(s)
Input
G(s)
C(s)
Output
System
(b)
R1(s)
+
+
−
R2(s)
R3(s)
C(s) = R1(s) + R2(s) − R3(s)
Summing junction
(c)
R(s)
R(s)
R(s)
R(s)
Pickoff point
(d)

CASCADE

- Any finite number of blocks in series may be algebraically combined by multiplication of transfer functions.
- That is, *n components or blocks with transfer functions* G_1 , G_2, . . . , G_n, *connected in cascade* are equivalent to a single element G with a transfer function given by

$$G = G_1 \cdot G_2 \cdot G_3 \cdots G_n = \prod_{i=1}^{n} G_i$$

Example

- Multiplication of transfer functions is *commutative; that is,*

$$GiGj = GjGi$$

for any i or *j* .

Parallel Form:

- Parallel subsystems have a common input and an output formed by the algebraic sum of the outputs from all of the subsystems.

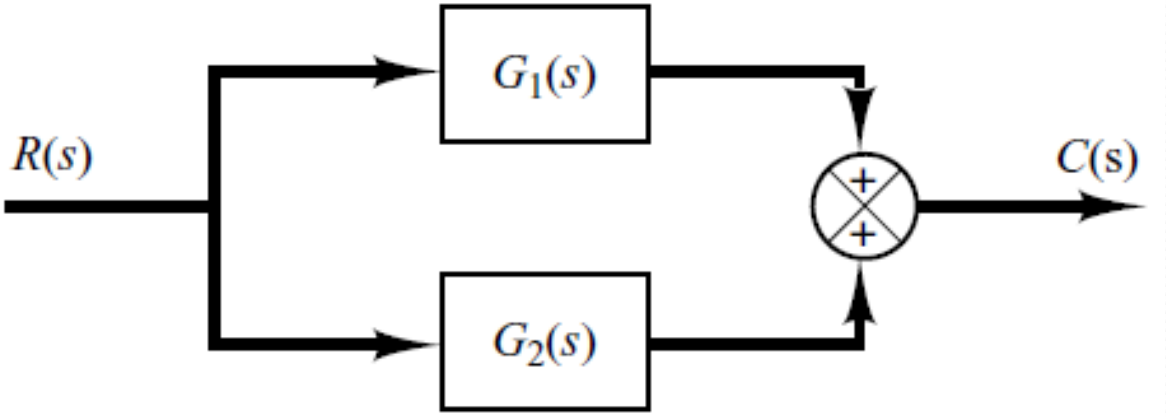


Figure: Parallel Subsystems.

Parallel Form:

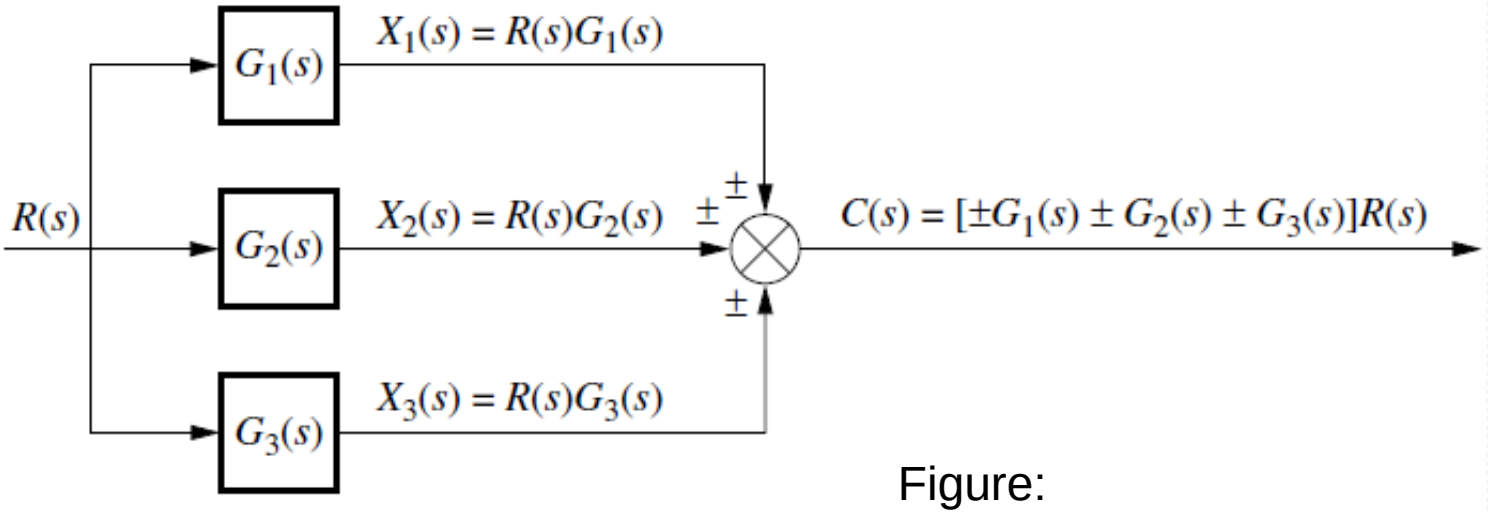


Figure:
a) Parallel Subsystems.
b) Equivalent Transfer Function.

The equivalent transfer function is

$$G_e(s) = \pm G_1(s) \pm G_2(s) \pm G_3(s)$$

Feedback Form:

- The third topology is the feedback form. Let us derive the transfer function that represents the system from its input to its output. The typical feedback system, shown in figure:

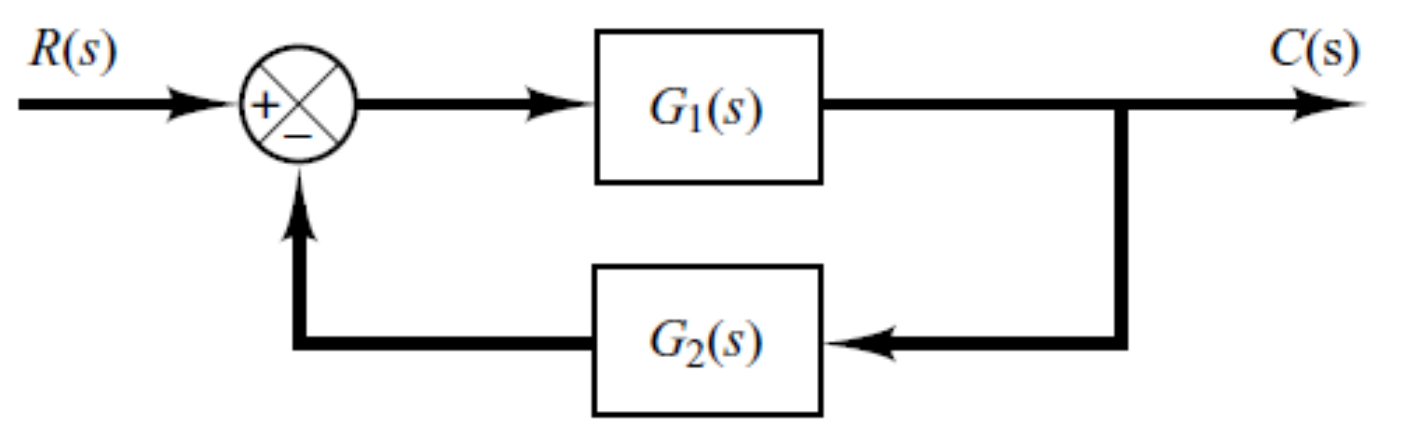


Figure: Feedback (Closed Loop) Control System.

The system is said to have negative feedback if the sign at the summing junction is negative and positive feedback if the sign is positive.

Feedback Form:

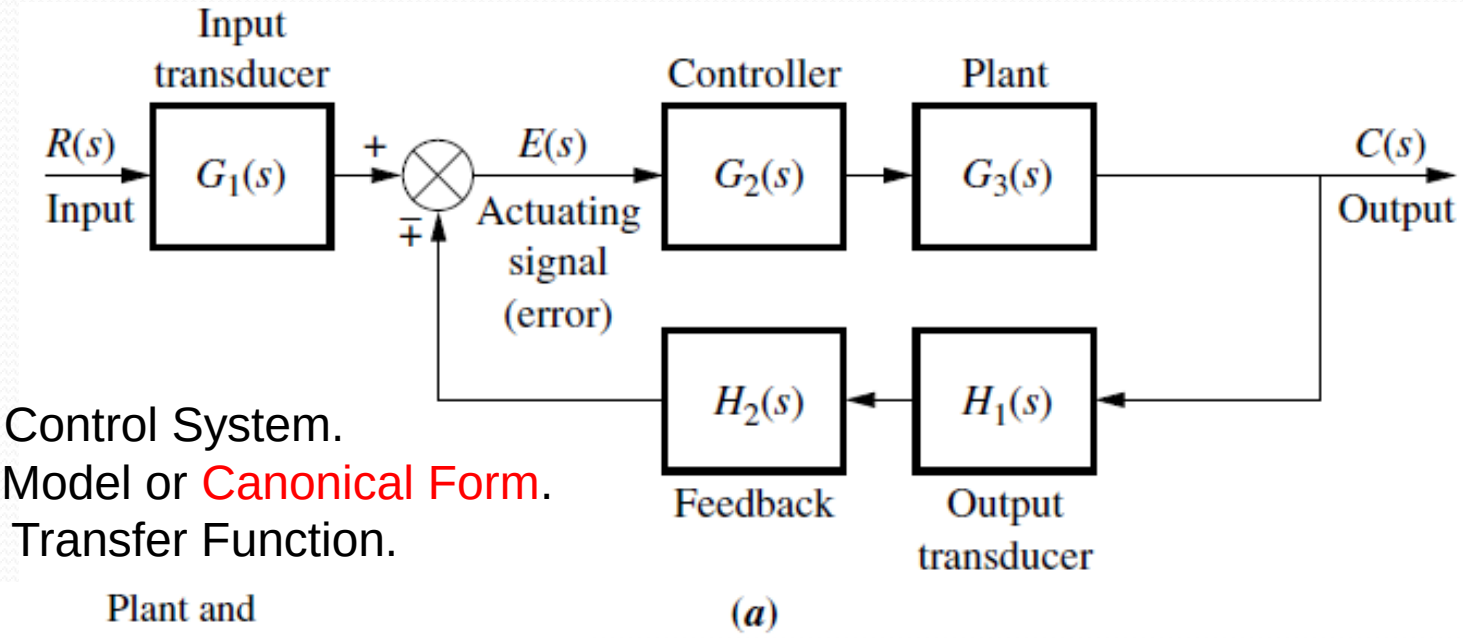
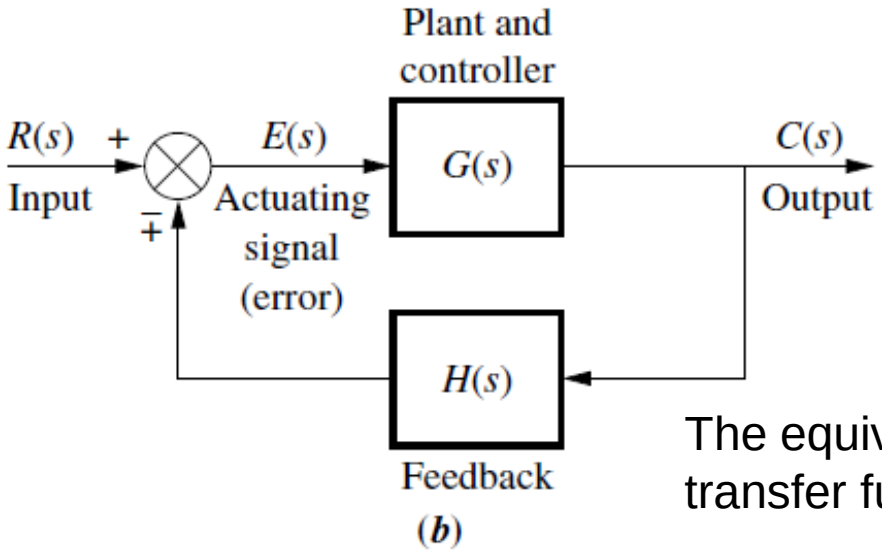
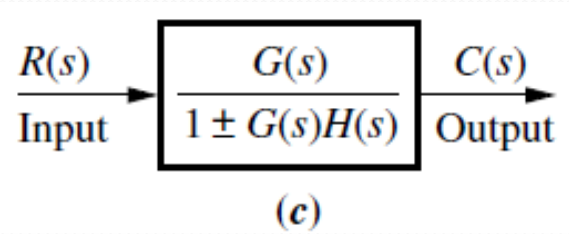


Figure:

a) Feedback Control System.
b) Simplified Model or Canonical Form.
c) Equivalent Transfer Function.



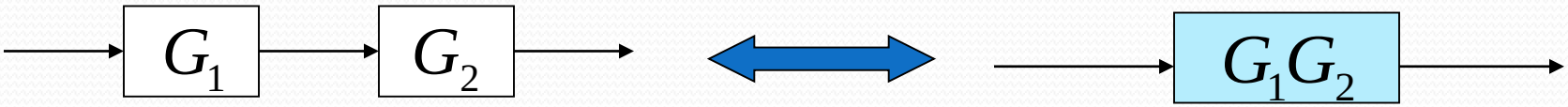


The equivalent or closed-loop transfer function is

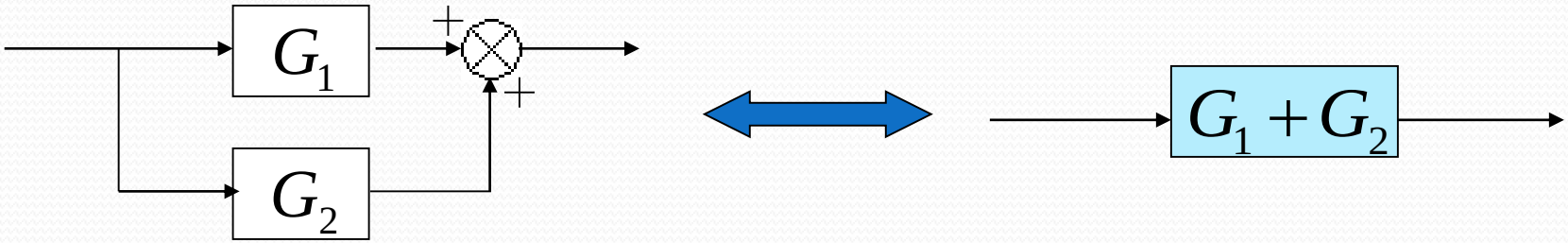
$$G_e(s) = \frac{G(s)}{1 \pm G(s)H(s)}$$

Reduction techniques

1. Combining blocks in cascade

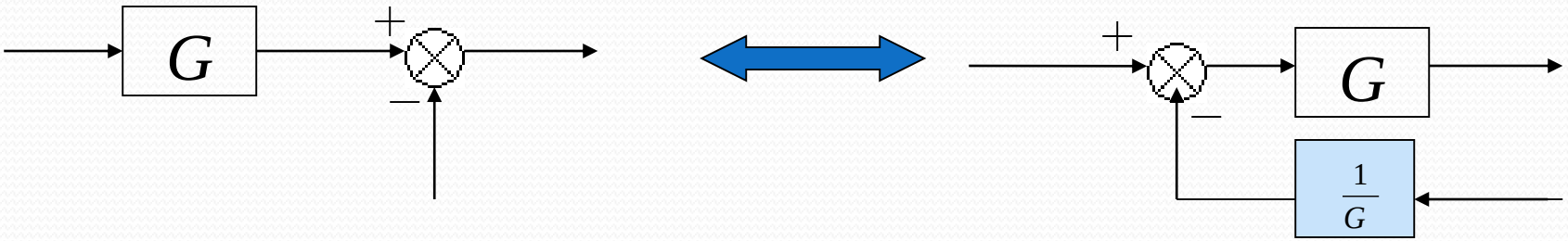


2. Combining blocks in parallel

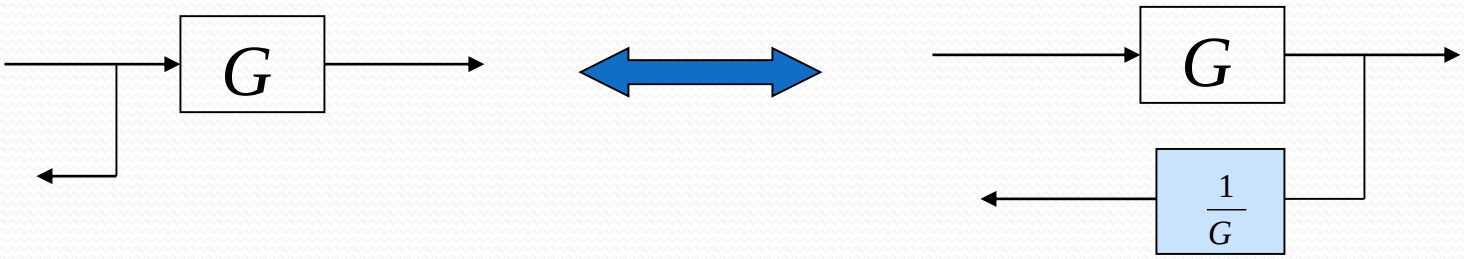


Reduction techniques

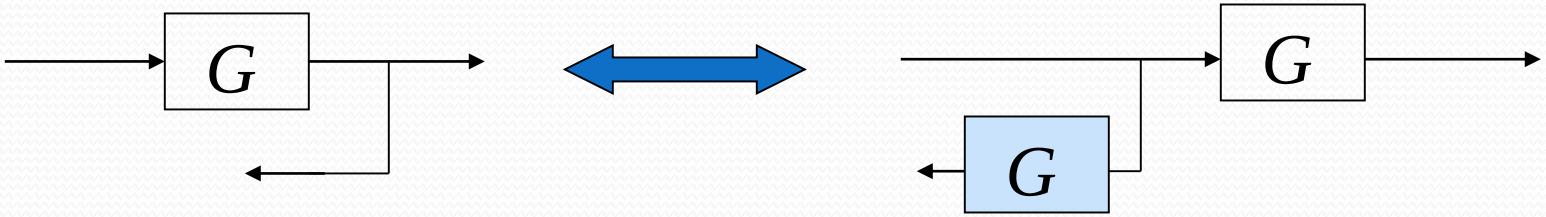
3. Moving a summing point ahead of a block



4. Moving a pickoff point behind a block

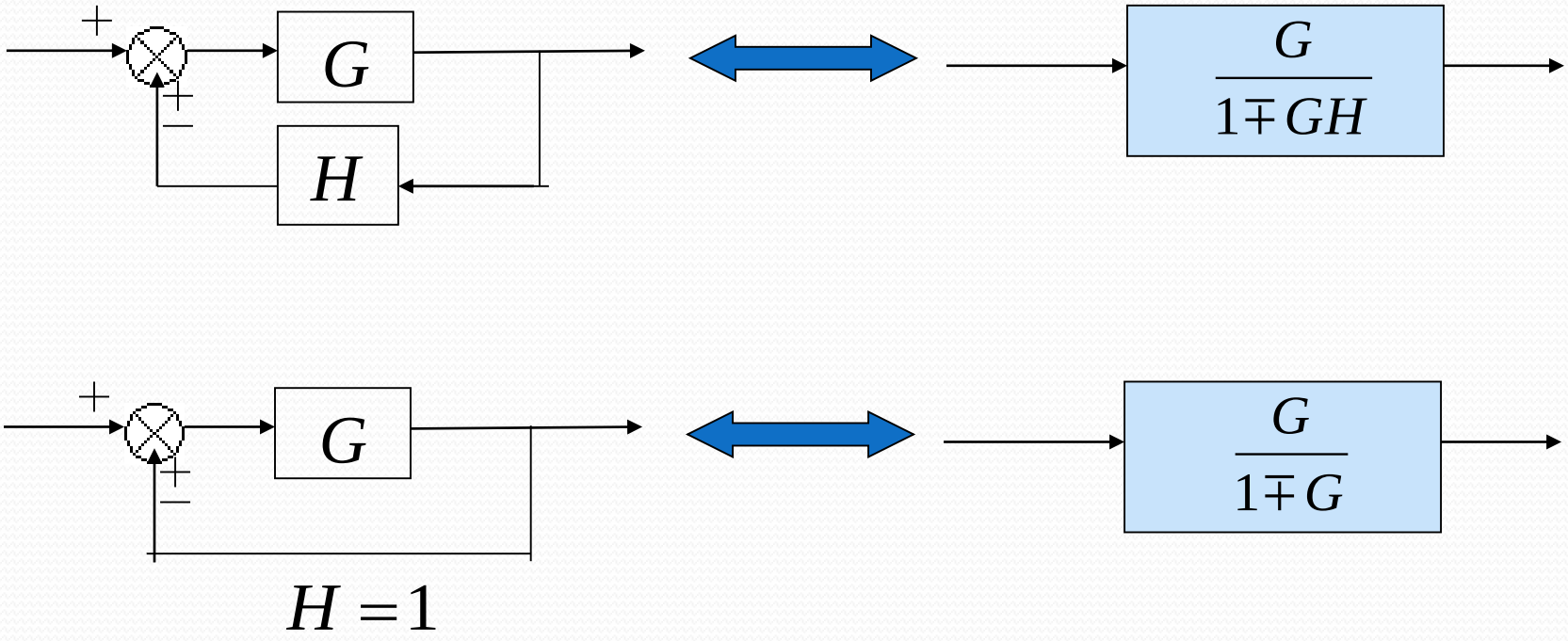


5. Moving a pickoff point ahead of a block

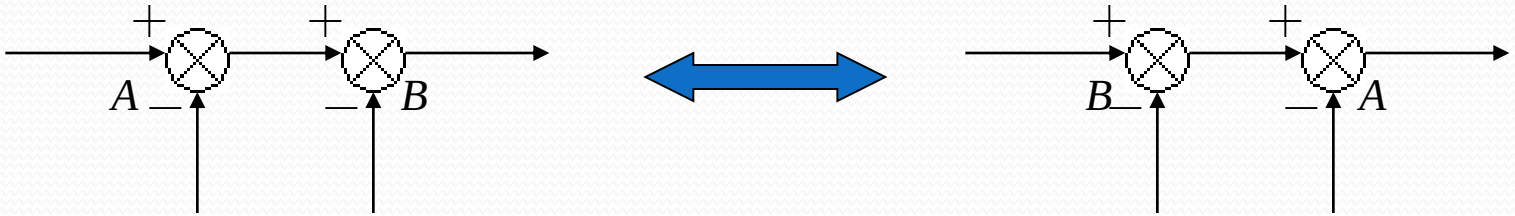


Reduction techniques

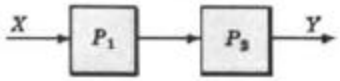
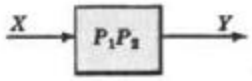
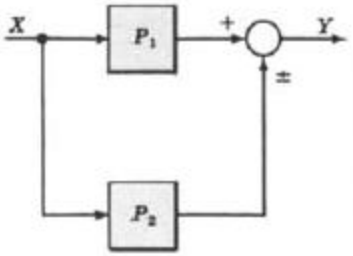
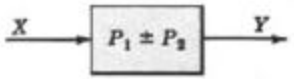
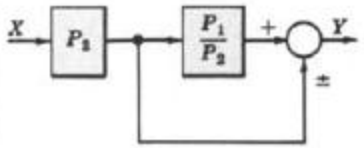
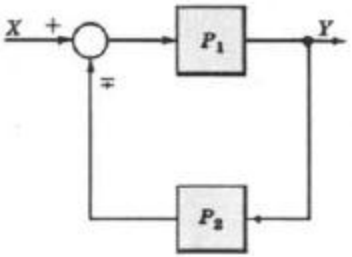
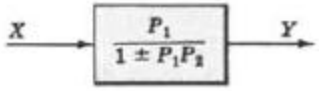
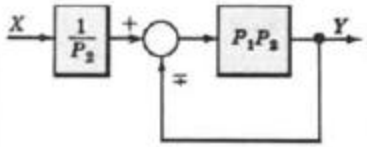
6. Eliminating a feedback loop



7. Swap with two neighboring summing points

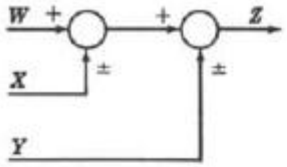
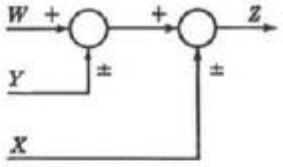
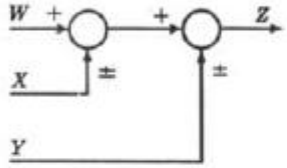
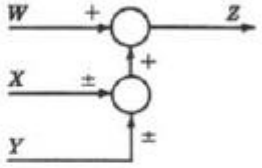
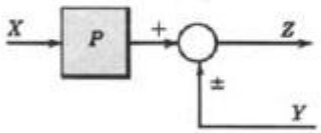
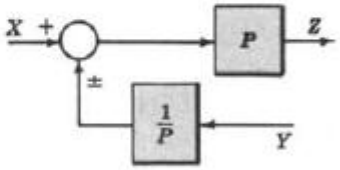
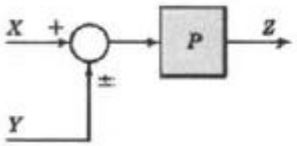
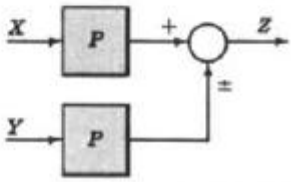


Block Diagram Transformation Theorems

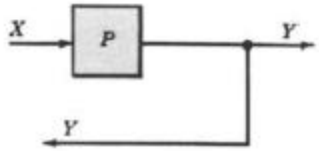
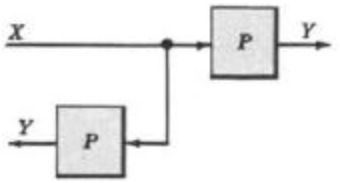
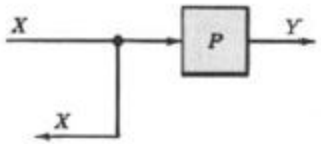
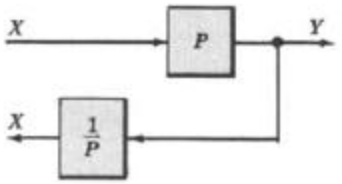
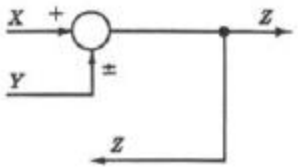
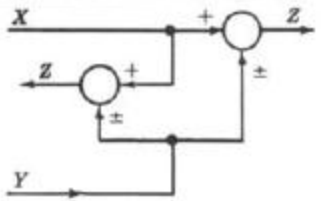
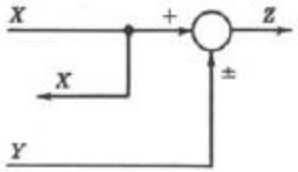
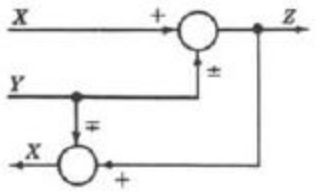
	Transformation	Equation	Block Diagram	Equivalent Block Diagram
1	Combining Blocks in Cascade	$Y = (P_1P_2)X$		
2	Combining Blocks in Parallel; or Eliminating a Forward Loop	$Y = P_1X \pm P_2X$		
3	Removing a Block from a Forward Path	$Y = P_1X \pm P_2X$		
4	Eliminating a Feedback Loop	$Y = P_1(X \mp P_2Y)$		
5	Removing a Block from a Feedback Loop	$Y = P_1(X \mp P_2Y)$		

The letter ***P is used to represent any transfer function, and W, X , Y, Z denote any transformed*** signals.

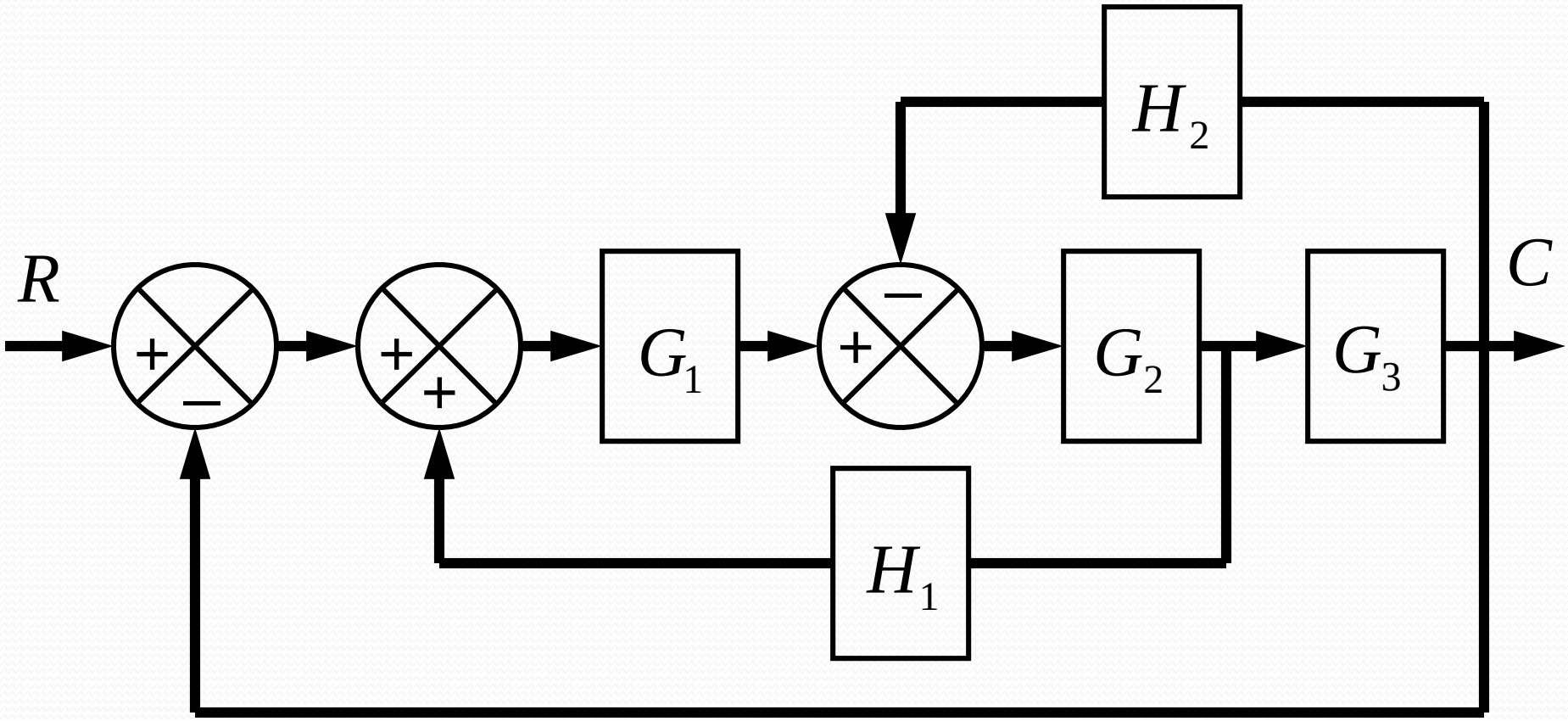
Transformation Theorems Continue:

	Transformation	Equation	Block Diagram	Equivalent Block Diagram
6a	Rearranging Summing Points	$Z = W \pm X \pm Y$		
6b	Rearranging Summing Points	$Z = W \pm X \pm Y$		
7	Moving a Summing Point Ahead of a Block	$Z = PX \pm Y$		
8	Moving a Summing Point Beyond a Block	$Z = P[X \pm Y]$		

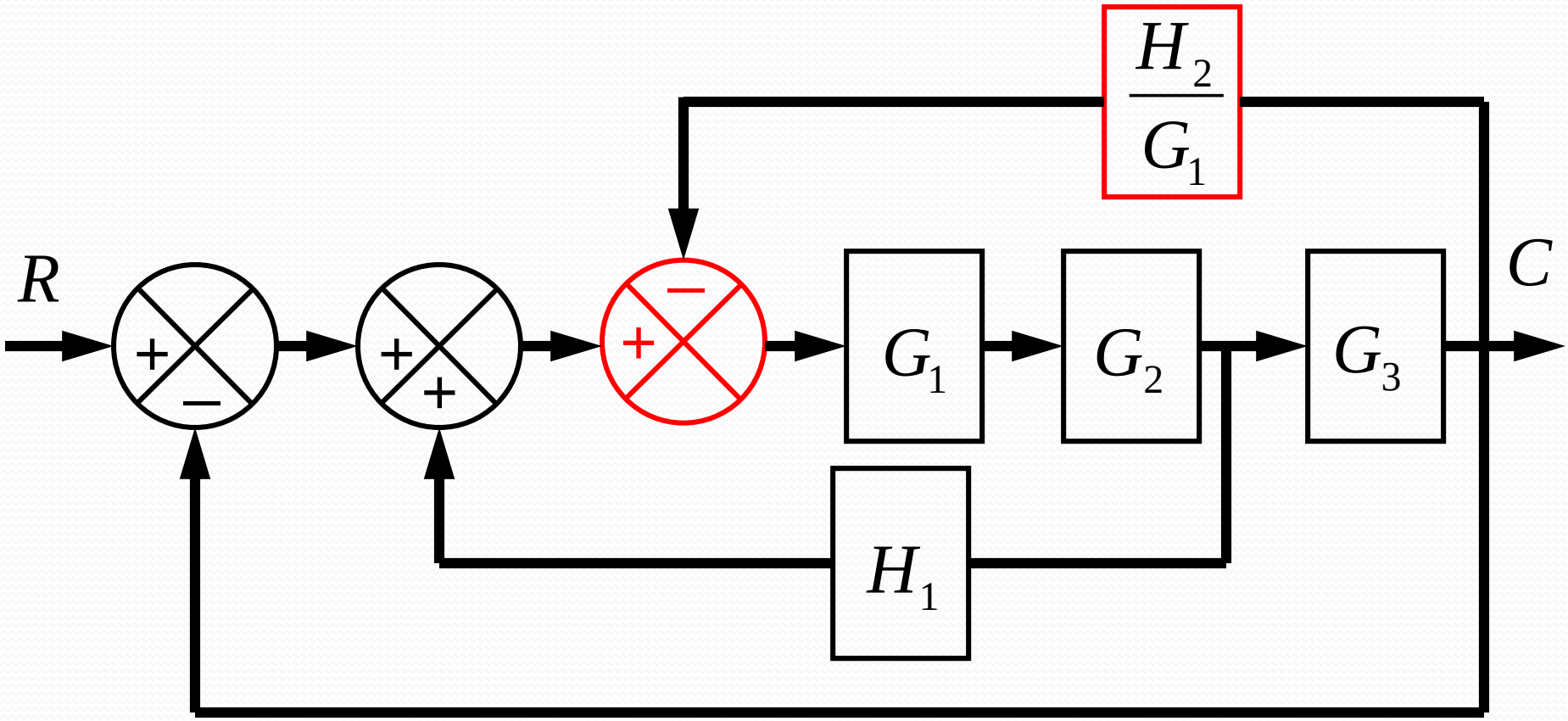
Transformation Theorems Continue:

	Transformation	Equation	Block Diagram	Equivalent Block Diagram
9	Moving a Takeoff Point Ahead of a Block	$Y = PX$		
10	Moving a Takeoff Point Beyond a Block	$Y = PX$		
11	Moving a Takeoff Point Ahead of a Summing Point	$Z = X \pm Y$		
12	Moving a Takeoff Point Beyond a Summing Point	$Z = X \pm Y$		

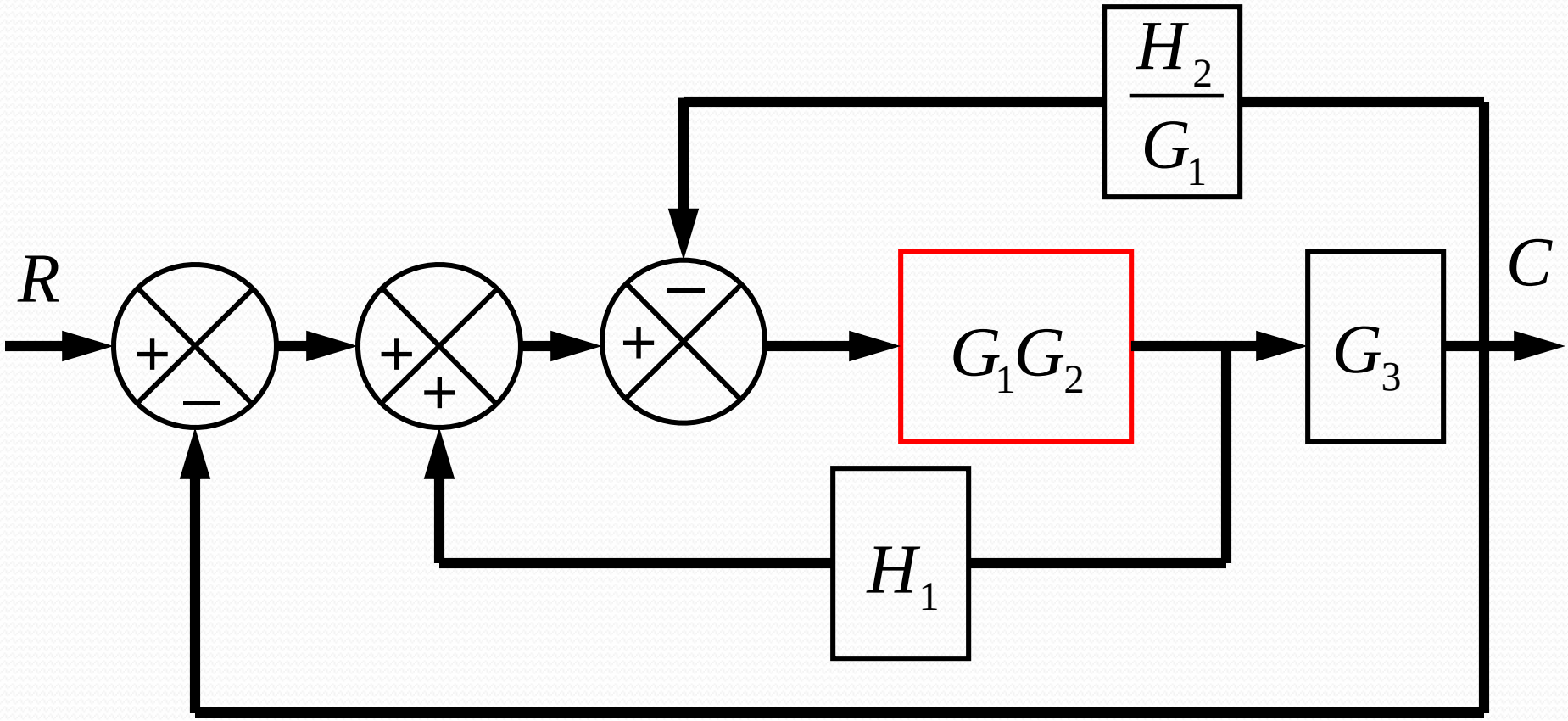
Example-1: Reduce the Block Diagram.



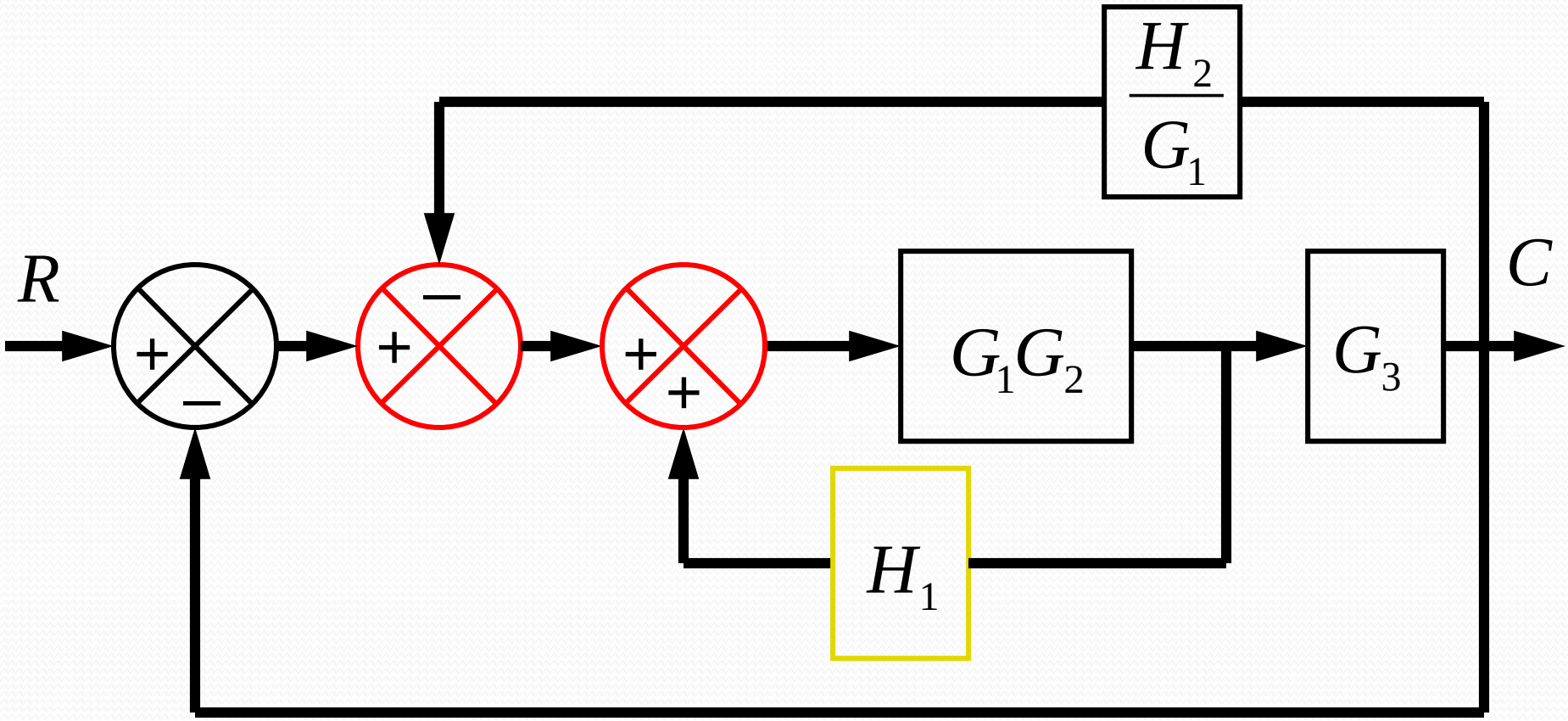
Example-1:



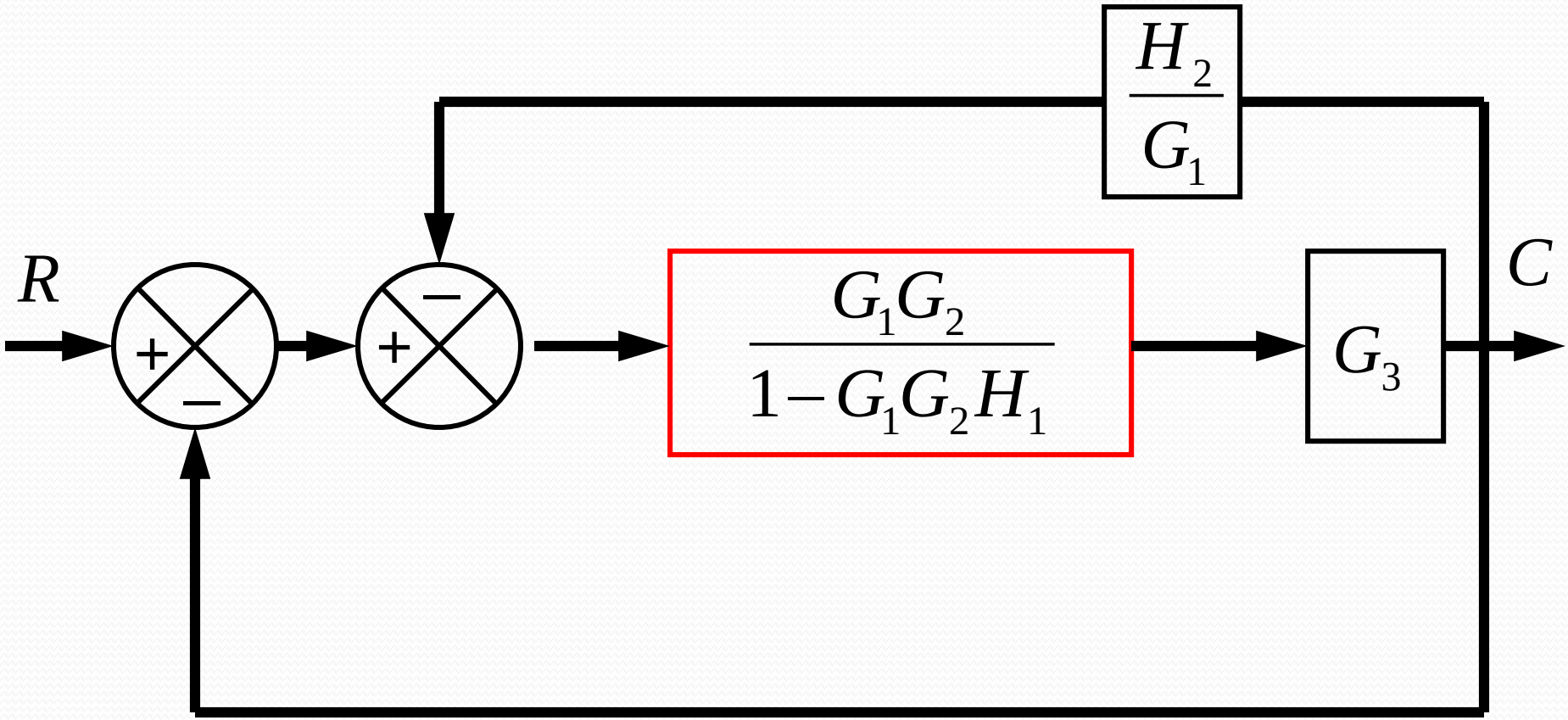
Example-1:



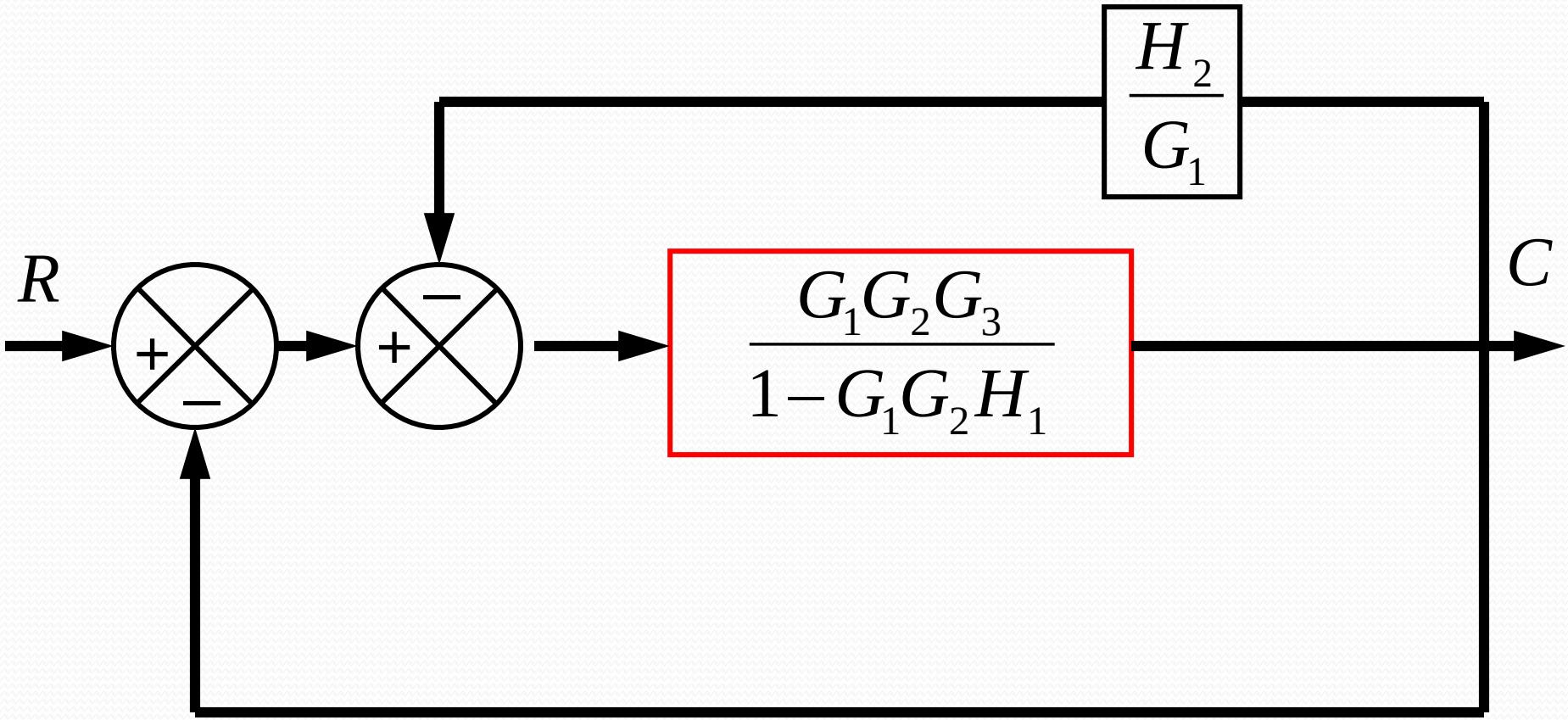
Example-1:



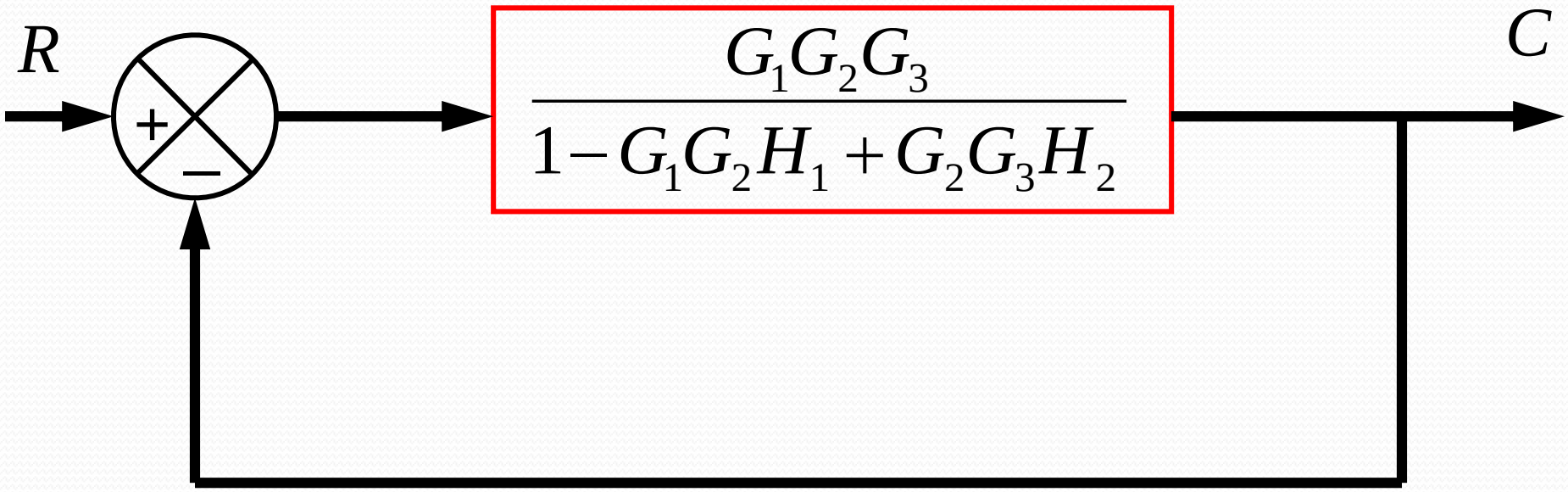
Example-1:



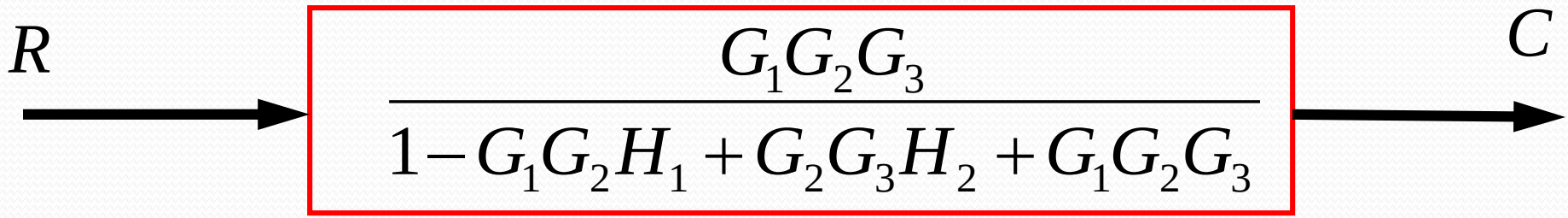
Example-1:



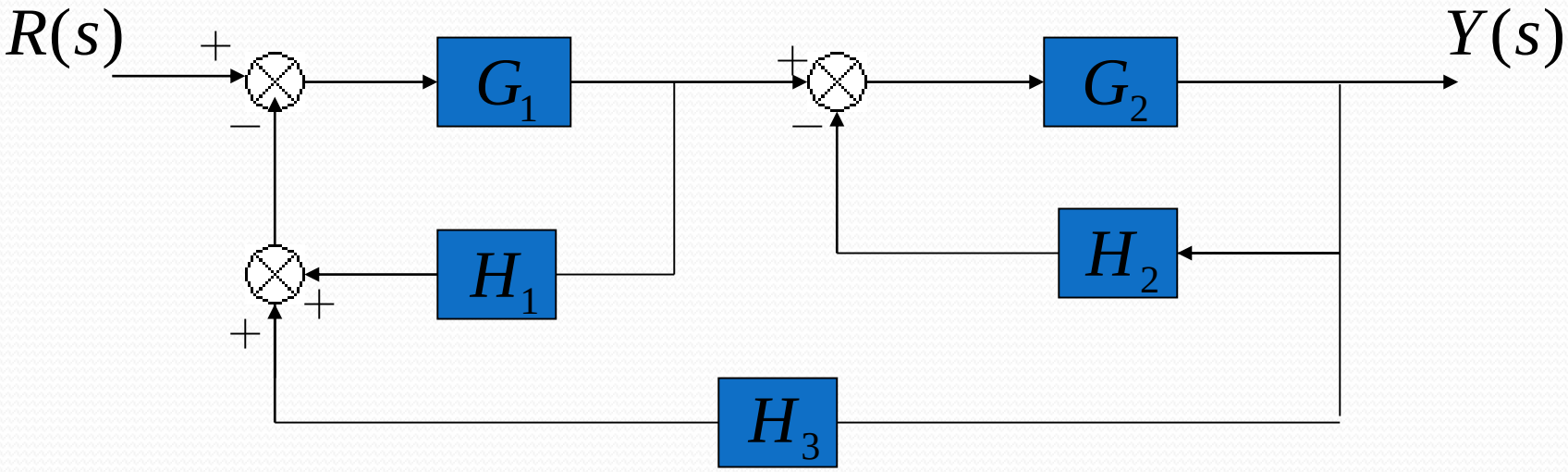
Example-1:



Example-1:

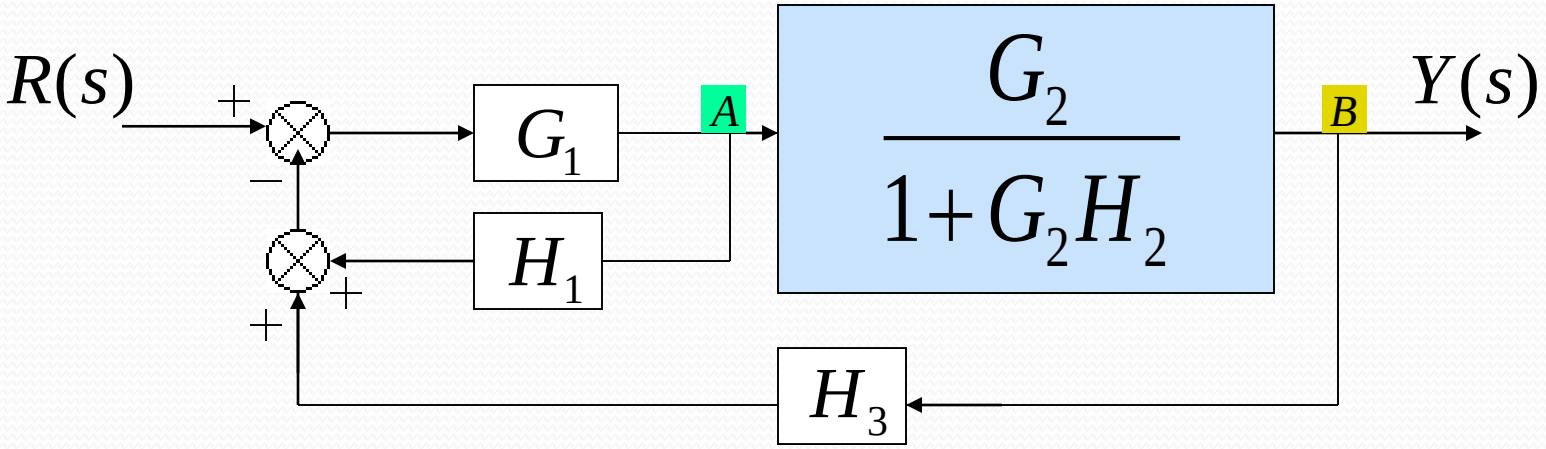


Example 2: Find the transfer function of the following block diagrams.

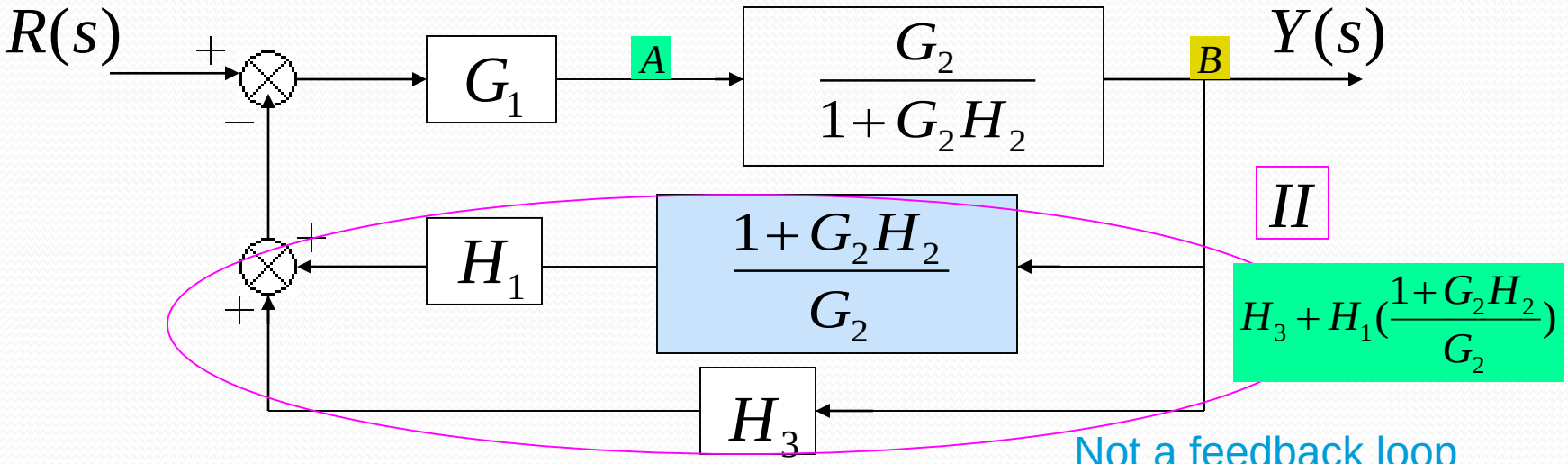


Solution:

1. Eliminate loop I

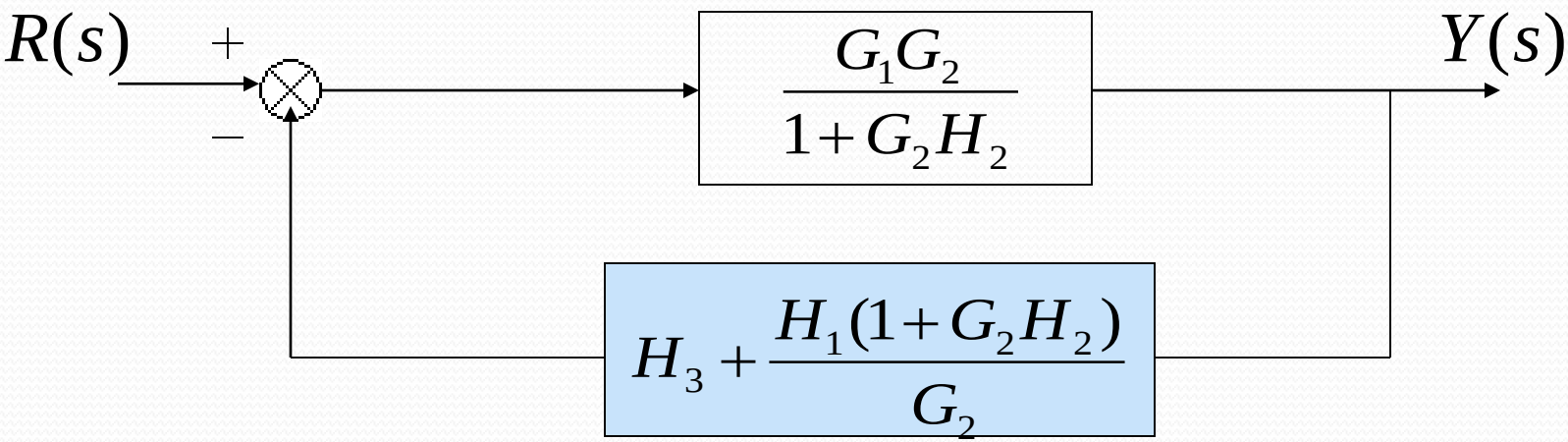


2. Moving pickoff point A behind block $\frac{G_2}{1+G_2H_2}$



Not a feedback loop

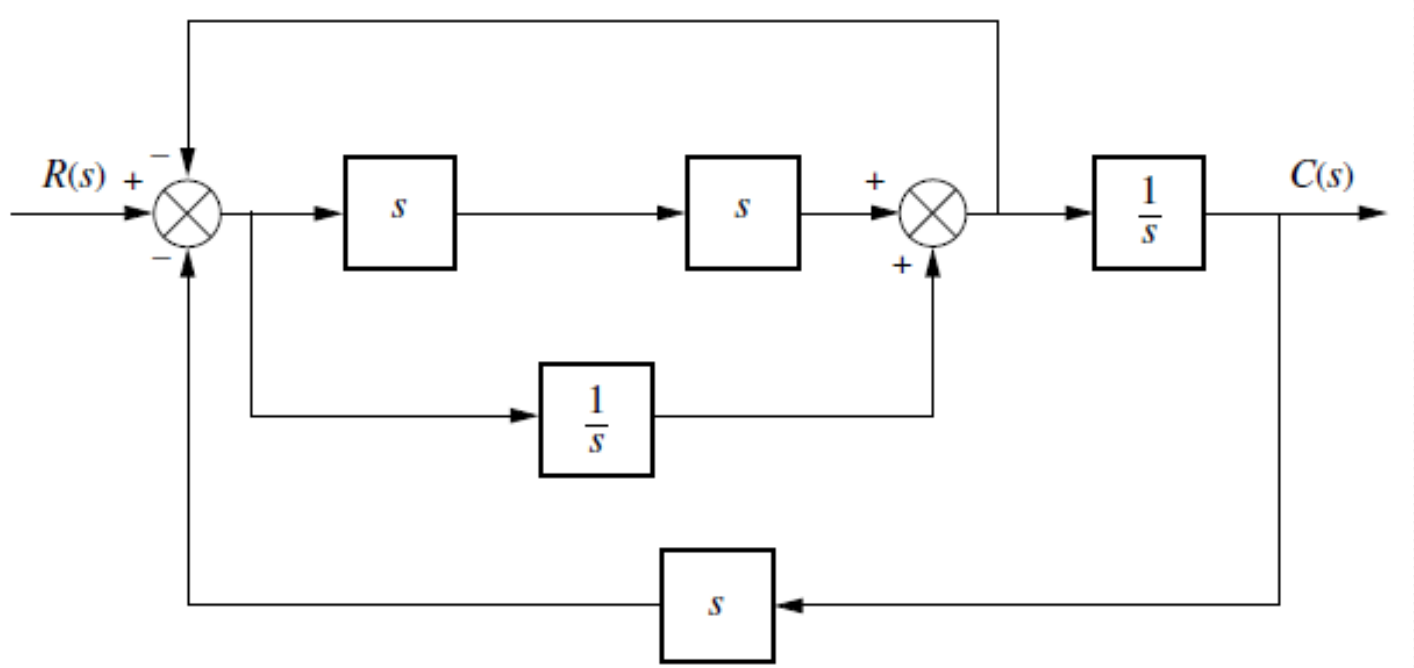
3. Eliminate loop II



$$\frac{Y(s)}{R(s)} = \frac{G_1 G_2}{1 + G_2 H_2 + G_1 G_2 H_3 + G_1 H_1 + G_1 G_2 H_1 H_2}$$

Skill Assessment Exercise:

PROBLEM: Find the equivalent transfer function, $T(s) = C(s)/R(s)$, for the system



Answer of Skill Assessment Exercise:

ANSWER: $$T(s) = \frac{s^3 + 1}{2s^4 + s^2 + 2s}$$