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Title: **Performance Alalysis of Solar Thermal Collectors**

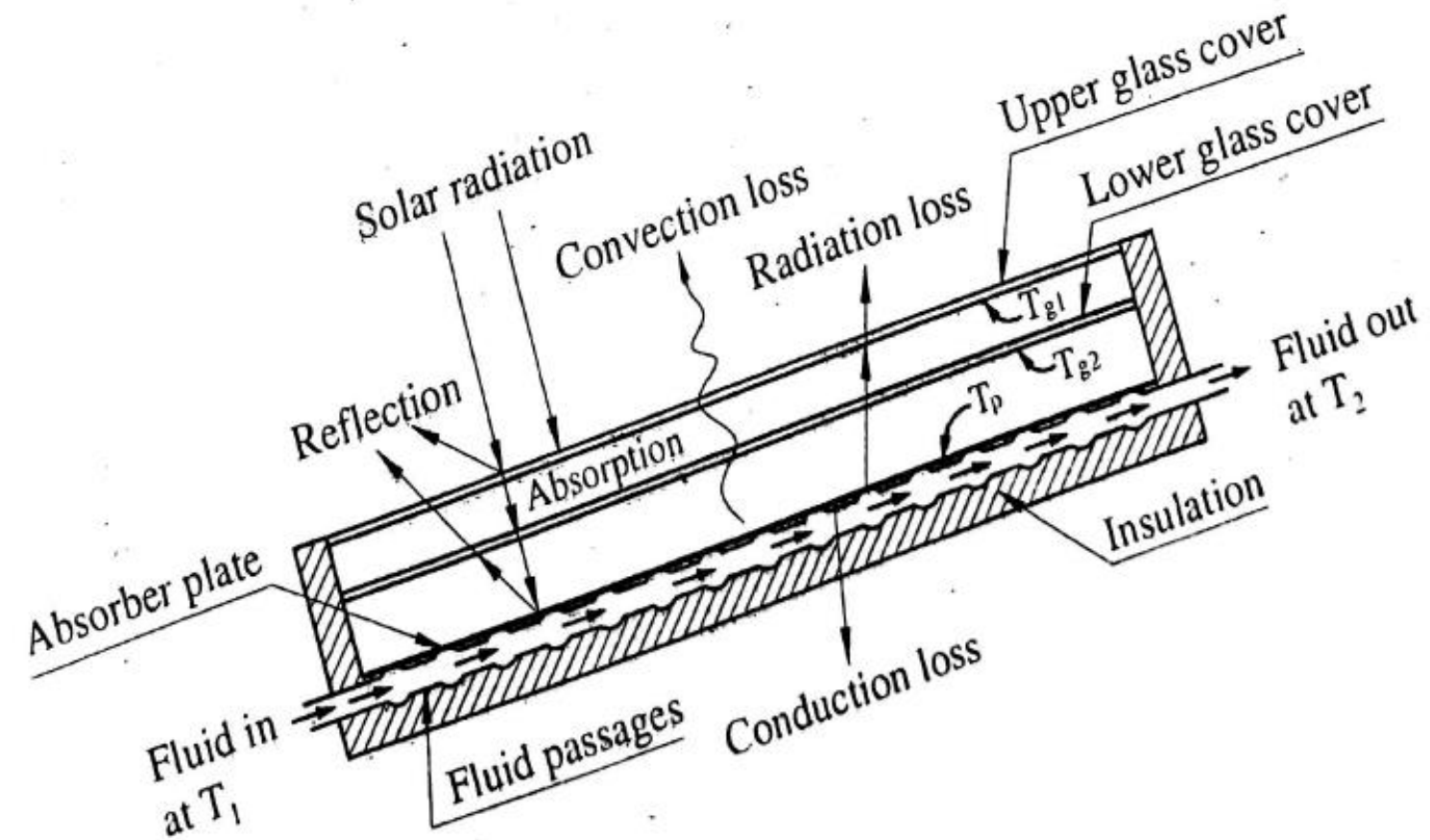
Date: 23/03/2020

Name of Faculty: Dr Hemant Kumar Gupta

Lecture No :01(10.30 to 11.30 am)

Source of information : Solar Energy (A book by S P Sukhatme)

# Energy balance equation of flat plate collector



Under steady state conditions, the useful heat delivered by a solar collector is equal to the energy absorbed in the metal surface minus the heat losses from the surface to surroundings. Mathematically,

$$H_T \cdot A_c \cdot (\tau\alpha)_e = Q_u + Q_L \quad \dots(2.21)$$

where,  $H_T$  = total solar radiation incident on the collector per unit area and time,  $\text{W/m}^2$

$A_c$  = collector area,  $\text{m}^2$

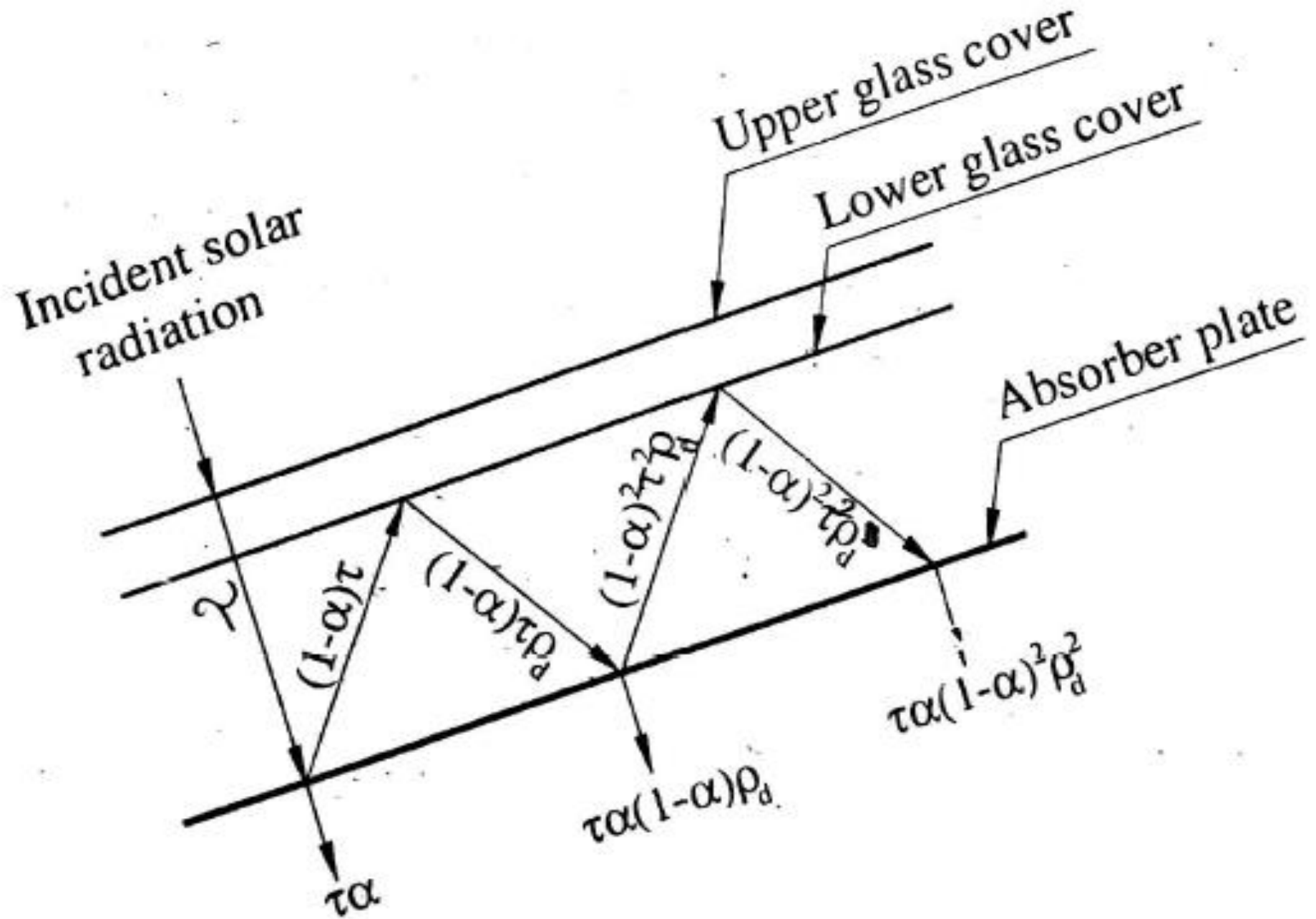
$Q_u$  = rate of useful heat collected from the collector, W

$Q_L$  = rate of heat lost from the collector, W

$(\tau\alpha)_e$  = effective transmittance - absorptance product of cover system for beam and diffused radiation

$\tau$  = fraction of incoming solar radiation that reaches the absorbing surface (transmissivity).

$\alpha$  = fraction of solar energy reaching the surface that is absorbed (absorptivity).



Total energy absorbed by absorber plate

$$= H_T [\tau\alpha + \tau\alpha(1-\alpha)\rho_d + T\alpha(1-\alpha)^2\rho_d^2 + \dots]$$

$$= H_T \left[ \frac{\tau\alpha}{1 - (1-\alpha)\rho_d} \right]$$

$$= H_T (\tau\alpha)_e, \text{ where } (\tau\alpha)_e = \frac{\tau\alpha}{1 - (1-\alpha)\rho_d}$$

Now, total heat loss from collector is given by

$$Q_L = A_c U_L (T_p - T_a)$$

where,  $U_L$  = overall heat loss co-efficient ( $\text{W/m}^2 \text{K}$ )

$T_p$  = average temperature of upper surface of the absorber plate

$T_a$  = atmospheric temperature.

Now, equation (2.21) becomes

$$H_T A_c (\tau\alpha)_e = Q_u + A_c U_L (T_p - T_a)$$

$$\therefore Q_u = A_c [H_T (\tau\alpha)_e - U_L (T_p - T_a)]$$

# Overall loss coefficient

The overall heat loss from the collector is difficult to determine since it depending on the mode of heat transfer (conduction, convection and radiation), it also depends on the temperature of plate, temperature of air, temperature of sky, temperature of cover system, number of glass cover, conductivity, thickness of rear and edge insulation, tilt of the collector, absorptivity and emissivity of absorber plate coating and emittance of glass cover. The rate of heat loss can be expressed as :

$$Q_L = U_L A_C (T_m - T_a) \quad \dots(2.30)$$

where,  $Q_L$  = rate of heat loss from collector

$U_L$  = overall heat loss co-efficient

$T_m$  = mean absorber plate temperature

$T_a$  = ambient temperature

The heat lost from the collector is the sum of the heat lost form the top, the bottom and the sides as :

$$Q_L = Q_t + Q_b + Q_s \quad \dots(2.31)$$



where  $Q_t$  = rate of heat loss from the top

$Q_b$  = rate of heat loss from the bottom

$Q_s$  = rate of heat loss from the sides

Each of these losses is also expressed in terms of co-efficients called the top loss co-efficient ( $U_t$ ), bottom loss co-efficient ( $U_b$ ) and side loss co-efficient ( $U_s$ ) as :

$$Q_t = U_t A_c (T_m - T_a)$$

$$Q_b = U_b A_c (T_m - T_a)$$

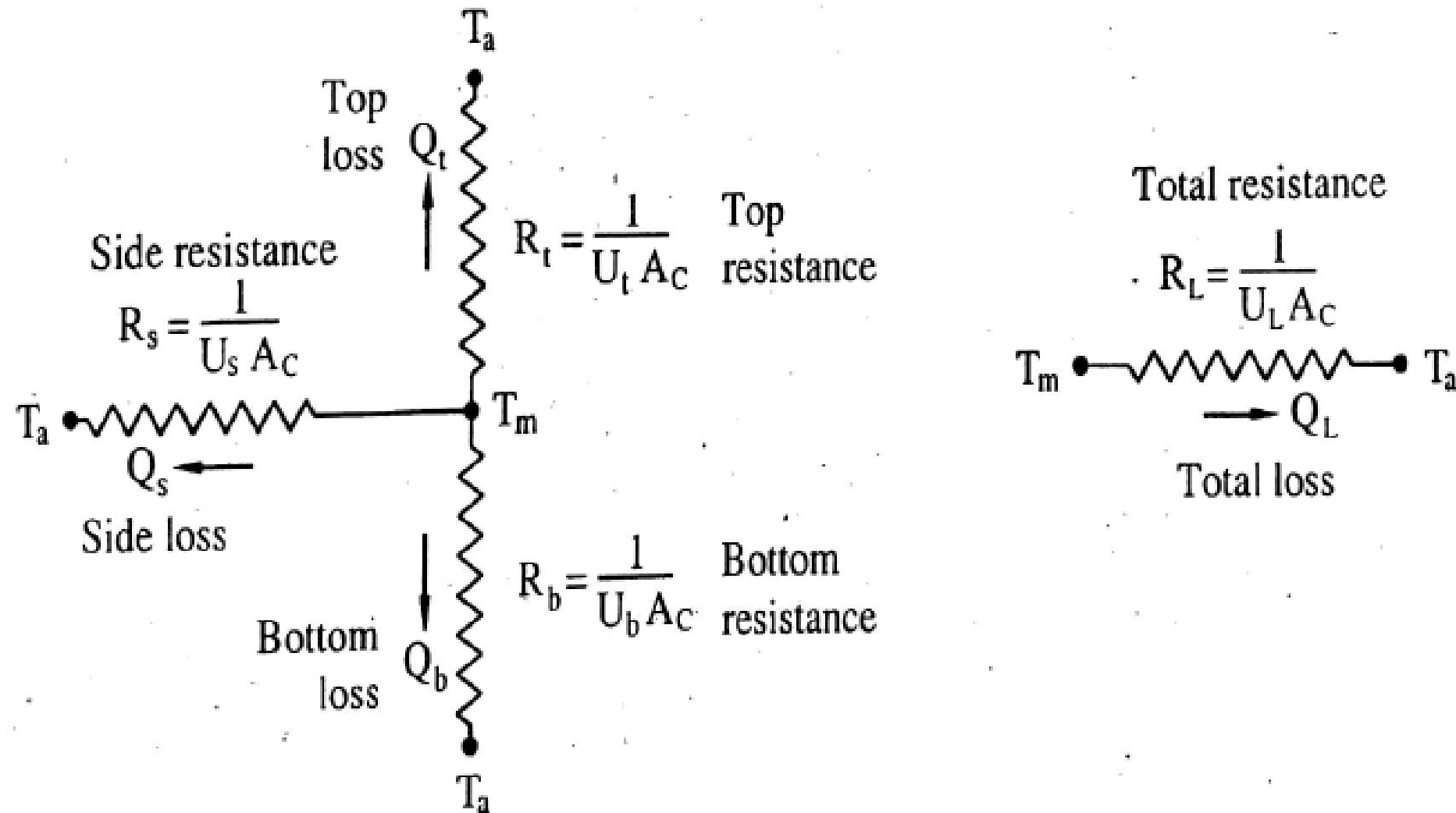
$$Q_s = U_s A_c (T_m - T_a) \quad \dots(2.32)$$

From equation (2.30), (2.31) and (2.32) we get

$$U_L A_c (T_m - T_a) = U_t A_c (T_m - T_a) + U_b A_c (T_m - T_a) + U_s A_c (T_m - T_a)$$

$$\therefore U_L = U_t + U_b + U_s$$

These losses can be expressed in terms of thermal resistance as shown in Fig. 2.32.

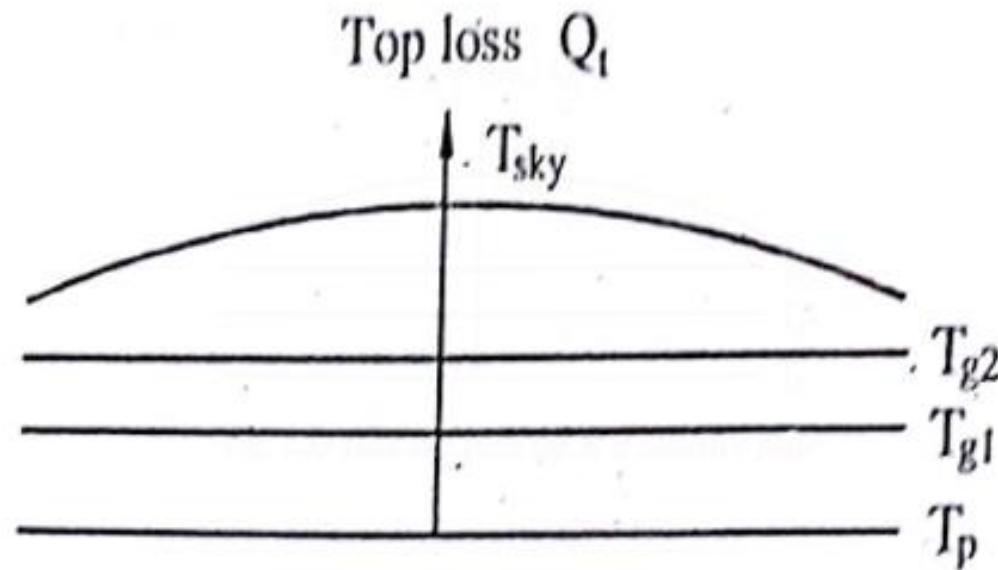


**Fig. 2.32 Overall heat loss from collector**



## Top Loss :

The heat loss from the top of collector is due to convection (both natural and forced) and radiation. The heat loss from the absorber plate first takes place from plate to the first glass cover, then first cover to second glass cover and so on last cover to the surroundings as shown in Fig. 2.33.



*Fig. 2.33 Heat loss from top of a collector plate*

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Hence, total heat loss from top of collector

$Q_t = \text{Convection loss} + \text{Radiation loss}$

$$= h_{pg_1} A_c (T_p - T_{g_1}) + \frac{A_c \sigma (T_p^4 - T_{g_1}^4)}{\left( \frac{1}{\epsilon_p} + \frac{1}{\epsilon_{g_1}} - 1 \right)}$$

(Convection and radiation heat transfer from absorber plate to first glass cover)

$$= h_{g_1 g_2} A_c (T_{g_1} - T_{g_2}) + \frac{A_c \sigma (T_{g_1}^4 - T_{g_2}^4)}{\left( \frac{1}{\epsilon_{g_1}} + \frac{1}{\epsilon_{g_2}} - 1 \right)}$$

(Convection and radiation heat transfer from first glass to second glass cover)

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$$\checkmark \quad = h_a A_c (T_{g2} - T_a) + \frac{A_c \sigma (T_{g2}^4 - T_{sky}^4)}{\left( \frac{1}{\epsilon_{g2}} + \frac{1}{\epsilon_{sky}} - 1 \right)}$$

(Convection and radiation heat transfer from second glass cover to surroundings)

where,

$h_{pg_1}$  = convective heat transfer co-efficient between the absorber plate and first glass cover

$h_{g_1g_2}$  = convective heat transfer co-efficient between the first glass cover and second glass cover

$h_a$  = convective heat transfer co-efficient between second glass cover and surroundings

$T_p$  = temperature of absorber plate

$T_{g_1}$  = temperature of first glass cover

$T_{g_2}$  = temperature of second glass cover

$T_a$  = temperature of atmospheric air

$T_{sky}$  = effective temperature of the sky with which the radiative heat exchange takes place  
 $= T_a - 6$

$\epsilon_p$  = emissivity of the absorber plate

$\epsilon_{g_1}, \epsilon_{g_2}$  = emissivities of first and second glass covers

The convective heat transfer co-efficients  $h_{pg}$ , and  $h_{g_1g_2}$  are calculated with help of equation of Nusselt Number of natural convection between two parallel surfaces. The convective heat transfer co-efficient  $h_a$  is calculated with help of Nusselt Number of forced convection, since outside wind having certain velocity. Generally,  $h_a = 2.8 + 3V$ , where  $V$  = velocity of wind (m/s) is taken.

The top loss co-efficient can be calculated by following equation.

$$U_T = \left[ \frac{n}{\left( \frac{C}{T_m} \right) \left( \frac{T_m - T_a}{n + f} \right)^{0.252}} + \frac{1}{h_a} \right]^{-1} + \left[ \frac{\sigma(T_m^2 - T_a^2)(T_m + T_a)}{\frac{1}{\epsilon_p + 0.0425n(1 - \epsilon_p)} + \frac{2n + f - 1}{\epsilon_g} - n} \right] \dots(2.33)$$

where,  $n$  = no. of glass covers

$$f = \left( \frac{9}{h_a} - \frac{30}{h_a^2} \right) \left( \frac{T_a}{316.9} \right) (1 + 0.09 \ln)$$

$$C = \frac{204.429(\cos \beta)^{0.252}}{L^{0.24}}$$

$\beta$  = collector tilt from horizontal

$L$  = spacing between plates

## Bottom loss :

The heat loss from the bottom of collector is due to conduction, convection and radiation. In most of cases, the thickness of insulation provided is such that the thermal resistance associated with conduction dominates, and hence assuming heat transfer due to convection

is negligible.

$$\begin{aligned} Q_b &= k_i A_c \frac{dt}{dx} \\ &= k_i A_c \frac{(T_m - T_a)}{\delta_i} \end{aligned}$$

where,  $k_i$  = thermal conductivity of insulation,  $\delta_i$  = thickness of insulation

$$= \frac{k_i}{\delta_i} A_c (T_m - T_a)$$

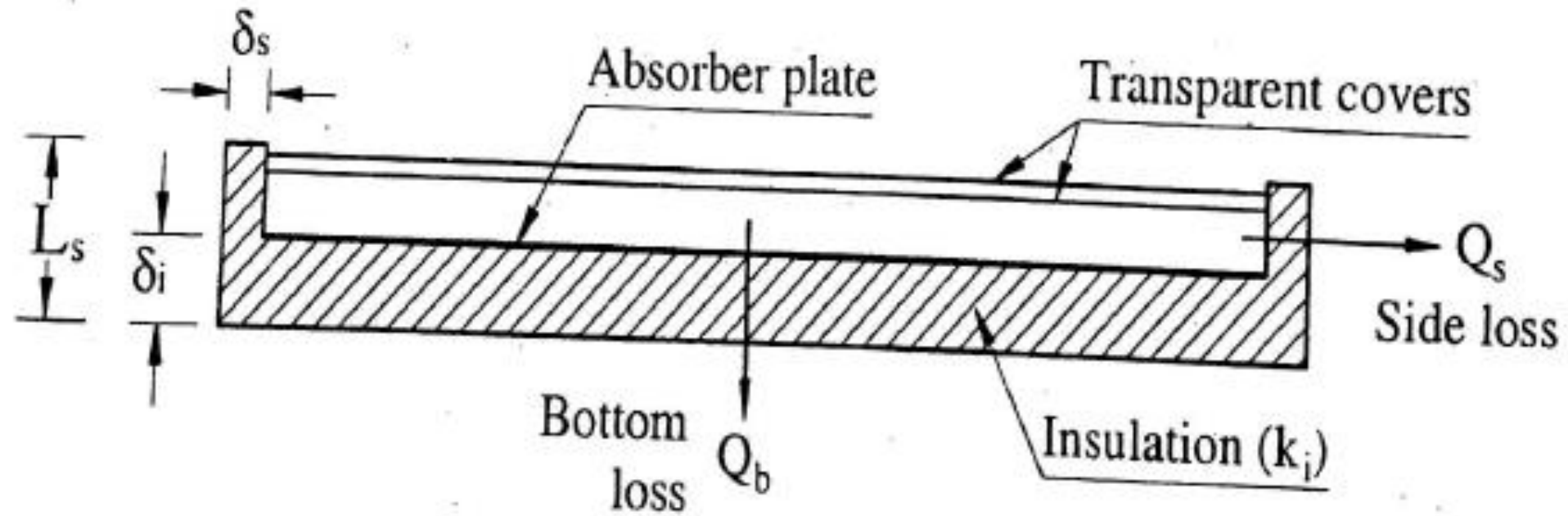
$$= U_b A_c (T_m - T_a)$$



Hence, bottom loss co-efficient

$$U_b = \frac{k_i}{\delta_i}$$

...(2.34)



*Fig. 2.34 Bottom and side losses from collector*

### Side loss :

As in case of the bottom loss co-efficient, it will assumed that the conduction resistance dominates and that the flow of heat is one dimensional and steady. The one dimensional approximation can be justified on the grounds that the side loss co-efficient is always much smaller, than the top loss co-efficient.

$$Q_s = 2L_s (L_1 + L_2) k_i \frac{T_m - T_a}{2\delta_s}$$

$$\text{Hence, } U_s = \frac{(L_1 + L_2)L_3}{L_1 L_2} \frac{k_i}{\delta_s} \quad \dots(2.35)$$

where,  $L_1$  and  $L_2$  is dimensions of absorber plate

# Collector efficiency

Collector efficiency  $\eta_c$  is defined as the ratio of the useful heat gain over a time period to the incident solar radiation on the collector over the same time period. Thus the **instantaneous collector efficiency** is given by:

$$\eta_i = \frac{\text{Useful heat gain}}{\text{Incident solar radiation on the collector}} = \frac{Q_u}{A_a I_t} \quad \dots(3.27)$$

The useful heat gain  $Q_u$  is given by eqn. (3.9) as:

$$\frac{Q_u}{A_a} = [I_t(\tau\alpha)_e - U_l(T_p - T_a)] \quad \dots(3.28)$$

where the average absorber plate temperature  $T_p$  is generally not known. Therefore, a parameter called collector efficiency factor  $F'$  is introduced in the above equation to determine collector performance without  $T_p$ .  $F'$  is defined as the ratio of actual useful energy gain per riser tube per unit length to the useful energy gain if the entire absorber plate is at the local fluid temperature  $T_f$ . Hence eqn. (3.28) can be written as:

$$\frac{Q_u}{A_a} = F' [I_t(\tau\alpha)_e - U_l(T_f - T_a)] \quad \dots(3.29)$$

where  $T_f = (T_i + T_o)/2$ , and  $T_i$  and  $T_o$  are the inlet and outlet fluid temperatures.

The plate efficiency factor  $F'$  is expressed as [5]:

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The plate efficiency factor  $F'$  is expressed as [5]:

$$F' = \frac{1}{\left[ WU_l \left( \frac{1}{U_l[W - D_o]F + D_o} + \frac{\delta_a}{k_a D_o} + \frac{1}{\pi D_i h_f} \right) \right]} \quad \dots(3.30)$$

where  $W$  is the tube spacing, *i.e.*, the centre to centre distance between the riser tubes

$D_i$  and  $D_o$  are the inner and outer diameters of the riser tubes

$\delta_a$  is the average thickness of the adhesive (bonding) material

$k_a$  is the thermal conductivity of the adhesive (bonding) material

$h_f$  is the heat transfer coefficient on the inside of the tube

$F$  is the fin efficiency factor and is given by:

$$F = \frac{\tanh[C(W - D_o)/2]}{[C(W - D_o)/2]}$$

where  $C = \left( \frac{U_l}{k_p \delta_p} \right)^{1/2}$

$k_p$  is the thermal conductivity of the absorber plate material

$\delta_p$  is the thickness of the absorber plate.

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Thus the instantaneous efficiency of the flat-plate collector in terms of mean fluid temperature is expressed as:

$$\eta_i = F' (\tau\alpha)_e - F' U_l \frac{(T_f - T_a)}{I_t} \quad \dots(3.31)$$

Also, another parameter  $F_R$ , called the collector heat removal factor, can be introduced in the eqns. (3.29) and (3.31) in place of  $F'$ . The  $F_R$  is defined as the ratio of actual useful heat energy gain to the useful energy gain if the entire absorber plate is at the temperature of the fluid entering the collector. Thus,

$$\frac{Q_u}{A_a} = F_R [I_t (\tau\alpha)_e - U_l (T_i - T_a)] \quad \dots(3.32)$$

and

$$\eta_i = F_R(\tau\alpha)_e - F_R U_l \frac{(T_i - T_a)}{I_t} \quad \dots(3.33)$$

where  $T_i$  is the inlet temperature of the fluid entering the collector.

The heat removal factor  $F_R$  is expressed as:

$$F_R = \frac{\dot{m}C_p}{U_l A_a} \left[ 1 - \exp \left( - \frac{U_l A_a F'}{\dot{m}C_p} \right) \right] \quad \dots(3.34)$$

where  $\dot{m}$  is the mass flow rate of the fluid (kg/s) and  $C_p$  is the specific heat of the fluid (kJ/kg K).  $F_R$  is the measure of the thermal resistance encountered by the absorbed solar radiation.

Equation (3.34) indicates that a straight line is obtained with a slope of  $F_R U_l$  and an intercept of  $F_R(\tau\alpha)_e$  if  $\eta_i$  is plotted against  $(T_i - T_a)/I_t$ . The actual performances of solar collectors are presented in this form. The procedure for performance testing has been proposed by the ASHRAE Standard 93-77 and IS 12933 (Part 5) 1990. Figure 3.4 presents the performance of typical liquid flat-plate collectors.

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# Assignment

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1. Derive the expression of collector efficiency factor, heat removal factor and useful heat gain for flat plate & cylindrical parabolic Solar Collector.
2. Derive the expression for governing energy balance equation for Solar Collector.

Hard copy submission date (31/3/2020)

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