Name of Topic : Buckingham π Theorem

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Name of Faculty: Mr. Ankursinh Solanki & Mr. Mitesh Gohil

Lecture No :01 (9:30 to 10:30)

Source of information : Fluid Mechanics and Hydraulic Machines by H.G. Katariay & J. P. Hadiya

BUCKINGHAM π THEOREM

Rayleigh method is not helpful when the number of independent variables is more than three or four. This difficulty is eliminated in Buckingham π Theorem

It states that if there are ‘n’ variables in a dimensionally homogeneous equation and if these variables contain 'm' fundamental dimensions (M, L, T), then they are grouped into (n - m), dimensionless independent π-terms.

Let $X_1, X_2, X_3, \ldots, X_n$ are the variables involved in a physical phenomenon. Let X_1 be the dependent variables and $X_2, X_3, \ldots X_n$ are the independent variables on which X_1 depends. Then X_1 is a function of $X_2, X_3, \ldots X_n$ and mathematically, it is expressed as

$$X_1 = f(X_2, X_3, \ldots X_n) \quad \ldots\ldots\ldots \quad (1)$$

Equation (1) can also be written as

$$F_1(X_1, X_2, X_3, \ldots X_n) = 0 \quad \ldots\ldots\ldots \quad (2)$$

This equation is a dimensionally homogeneous equation. It contains *n* variables. If there are *'m'* fundamental dimensions then according to Buckingham-π-theorem, equation (2) can be written interms in which number of π -terms is equal to (n - *m)*. Hence, equation (2) becomes

$$F(\pi_1, \pi_2, \ldots \pi_{n-m}) = 0 \qquad \ldots\ldots\ldots (3)$$

- Each of π terms is dimensionless and independent of the system.
- Each of π term contains (m +1) variables, where *m* is the number of Fundamental dimensions and is also called *repeating variables.*

SELECTION OF REPEATING VARIABLES

There is no separate rule for selecting repeating variables. Hut the number of repeating variables is equal to the fundamental dimensions of the problem. Generally, ρ, υ, l or ρ, υ, D) are choosen as repeating variables.

It means, one refers to fluid property (ρ), one refers to flow property (υ) and the other one refers to geometric property (l or D). In addition to this, the following points should be kept in mind while selecting the repeating variables:

1. No variables should be dimensionless.
2. The selected two repeating variables should not have the same dimensions.
3. The selected repeating variables should be independent as far as possible.

STEPS TO BE FOLLOWED IN BUCKINGHAM Π METHOD

1. First, the variables involved in a given analysis are listed to study about given phenomenon thoroughly.
2. Then, these variables are expressed in terms of primary dimensions.
3. Next, the repeating variables are chosen according to the hint given in selection of repeating variables. Once, the repeating variables should be checked either those are independent or dependent variables because all should be independent variables.
4. Then the dimensionless parameters are obtained by adding one at a time repeating variables.
5. The number of π-terms involved in dimensional analysis is calculated using, n - m = Number of π terms.

Where, n = Total number of variables involved in given analysis

m = Number of fundamental variables

6. Finally, each equation in exponential form is solved which means the coefficients of exponents are found by comparing both sides exponents. Then these dimensionless parameters are recombined and arranged suitably.

- In most of the fluid mechanics problems, the choice of repeating variables may be (i) d, v, ρ (ii) l, v, ρ (iii) l, v, μ or (iv) d, v, μ

Question: **The pressure difference Δp in a pipe of diameter D and length l due to the viscous flow depends on the velocity V, viscosity μ, density ρ. Using Buckingham's π –theorem, obtain the expression for Δp.**

Solution:

Δp is a function of D, l, V, μ, ρ, k

$\Delta p = f(D, l, V, \mu, \rho, k)$ (or) $f_1(\Delta p, D, l, V, \mu, \rho, k) = 0$ .... (i)

Hence total number of variables, n = 7.

Number of fundamental dimensions, m = 3.

Writing dimensions of each variable,

Dimension of, [$\Delta p = ML^{-1}T^{-2}$, $D = l$, $l = L$, $V = LT^{-1}$,

$\mu = ML^{-1}T^{-1}$ $\rho = ML^{-3}$, $k = L$]

Number of π-terms = n - m = 7 - 3 = 4.

Now equation (i), can be grouped in 4 π terms as, Equation (i) is written as $(\pi_1, \pi_2, \pi_3, \pi_4) = 0$ .... (ii)

Each π -term contains $(m + 1)$ variables or 3+1 variables, out of four variables, three are repeating variable. Choosing D, V and ρ as repeating variables, we have four π terms as,

$$\pi_1 = D^{a1} . V^{b1} . \rho^{c1} . \Delta p$$

$$\pi_2 = D^{a2} . V^{b2} . \rho^{c2} . l$$

$$\pi_3 = D^{a3} . V^{b3} . \rho^{c3} . \mu$$

$$\pi_4 = D^{a4} . V^{b4} . \rho^{c4} . k$$

First π – Term

$$\pi_1 = D^{a1} . V^{b1} . \rho^{c1} . \Delta p$$

Substituting dimensions on both sides of π_1,

$$M^0 L^0 T^0 = L^{a1} . (LT^{-1})^{b1} . (ML^{-3})^{c1} . ML^{-1} T^{-2}$$

Equating the powers of M, L, T on both sides

Power of *M*,	$0 = c_1 + 1,$	$\mathbf{c_1 = -1}$
Power of *L*,	$0 = a_1 + b_1 - 3c_1 - 1$	$\mathbf{a_1} = - b_1 + 3c_1 - 1 = 2-3+1 = 0$
Power of *T*,	$0 = - b_1 - 2,$	$\mathbf{b_1 = -2}$

Substituting the values of a_1, b_1 and c_1 in π_1, we get

$$\pi_1 = D^0 . V^{-2} . \rho^{-1} . \Delta p$$

$$\pi_1 = \frac{\Delta p}{\rho V^2}$$

Second π- Term

$$\pi_2 = D^{a2} . V^{b2} . \rho^{c2} . l$$

Substituting dimensions on both sides of π_2,

$$M^0 L^0 T^0 = L^{a2} . (LT^{-1})^{b2} . (ML^{-3})^{c2} . L$$

Equating the powers of M, L, T on both sides

Power of *M*,	$0 = c_2$,	$c_1 = 0$	$c_1 = 0$
Power of *L*,	$0 = a_2 - b_2 - 3c_2 + 1$,	$a_2 = b_2 + 3c_2 - 1 = -1$	$a_2 = -1$
Power of *T*,	$0 = -b_2$	$b_2 = 0$	$b_2 = 0$

Substituting the values of a_2, b_2 and c_2 in π_2, we get

$$\pi_2 = D^{-1} . V^0 . \rho^0 . l \quad = \frac{1}{D}$$

Third π - Term

$$\pi_3 = D^{a3} . V^{b3} . \rho^{c3} . \mu$$

Substituting dimensions on both sides of π_3,

$$M^0 L^0 T^0 = L^{a3} . (LT^{-1})^{b3} . (ML^{-3})^{c3} . ML^{-1} T^{-1}$$

Equating the powers of M, L, T on both sides

Power of *M*,	$0 = c_3 + 1,$	$c_3 = -1$	$c_3 = -1$
Power of *L*,	$0 = a_3 + b_3 - 3c_3 - 1,$	$a_3 = -b_3 + 3c_3 + 1 = 1-3+1 = -1$	$a_3 = -1$
Power of *T*,	$0 = -b_3 - 1,$	$b_3 = -1$	$b_3 = -1$

Substituting the values of a_3, b_3 and c_3 in π_3, we get

$$\pi_3 = D^{-1} . V^{-1} . \rho^{-1} . \mu = \frac{Q}{DV\rho}$$

Fourth π- Term

$$\pi_4 = D^{a4} . V^{b4} . \rho^{c4} . k$$

Substituting dimensions on both sides of π_3,

$$M^0 L^0 T^0 = L^{a4} . (LT^{-1})^{b4} . (ML^{-3})^{c4} . L$$

Equating the powers of M, L, T on both sides

Power of *M*,	$0 = c_4$,	$c_4 = 0$	$c_4 = 0$
Power of *L*,	$0 = a_4 - b_4 - 3c_4 + 1$,	$a_3 = b_4 + 3c_4 - 1 = -1$	$a_4 = -1$
Power of *T*,	$0 = -b_4$,	$b_4 = 0$	$b_4 = 0$

Substituting the values of a_3, b_3 and c_3 in π_4, we get

$$\pi_4 = D^{-1} . V^0 . P^0 . k = \frac{k}{D}$$

Substituting the values of π_1, π_2, π_3 and π_4 in equation (ii), we get

$$f_1\left(\frac{\Delta p}{\rho V^2}, \frac{l}{D}, \frac{Q}{DV\rho}, \frac{k}{D}\right) = 0$$

$$\frac{\Delta p}{\rho V^2} = \Phi\left(\frac{l}{D}, \frac{Q}{DV\rho}, \frac{k}{D}\right) \qquad \textbf{Ans}$$

Assignment

1. What are repeating variables? How are they selected for dimensional analysis?
2. Using Buckingham's π- theorem, show the efficiency η of a fan depends on density ρ, dynamic viscosity μ of the fluid, angular velocity ω, diameter D of the rotor and the discharge Q.

Note:- Kindly write the above assignment in Separate notebook and submit it on 31/3/2020.

- If any query regarding above topic kindly contact me on my mobile no.:-8758034606

Mr. Ankursinh Solanki
Mobile No. 8758034606

Thank You