⇒ # Divergence

The divergence of a vector point f^n $\bar{F}$ denoted by div $\bar{F}$ or $\nabla\cdot\bar{F}$ and is defined as,

$$\nabla\cdot\bar{F} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\bar{F}$$

where $\bar{F}$ is vector point f^n.

* Solenoidal Function.

A vector point f^n is said to be solenoidal if $\nabla\cdot\bar{F}=0$
For such a vector, there is no loss or gain of fluid.

⇒ # Curl

The curl of a vector point f^n $\bar{F}$ is denoted by curl $\bar{F}$ or $\nabla\times\bar{F}$ and is defined by

$$\nabla\times\bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

where $\bar{F} = (F_1\hat{i} + F_2\hat{j} + F_3\hat{k})$ {Here F_1, F_2, F_3 are component of $\bar{F}$}

* Irrotational Field.

A vector point f^n $\bar{F}$ is said to be irrotational if curl $\bar{F} = \bar{0}$

Note ① $\nabla\times\nabla\phi = 0$ {where ϕ is any scalar point f^n.}

② difference bet^n ~~scal~~ scalar and vector point f^n.

Consider the f^n. $f(x,y,z) = x^2 + yz$ and $F(x,y,z) = x^2\hat{i} + yz\hat{k}$

Here $f(x,y,z)$ is scalar point f^n as its gives us a ~~value~~ value
and $F(x,y,z)$ is vector point f^n. " " " " " vector

Q. ① If $\bar{A} = x^2 z\,\hat{i} - 2y^3 z^2\,\hat{j} + xy^2 z\,\hat{k}$, find $\nabla \cdot \bar{F}$ and curl $\bar{F}$ at the point $(1,-1,1)$

$$\nabla \cdot \bar{A} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left(x^2 z\hat{i} - 2y^3 z^2\hat{j} + xy^2 z\hat{k}\right)$$

$$\nabla \cdot \bar{A} = (2xz - 6y^2 z^2 + xy^2)$$

Now

$$(\nabla \cdot \bar{A})_{(1,-1,1)} = 2 - 6 + 1 = \underline{\underline{-3}}$$

*

Now

$$\nabla \times \bar{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 z & -2y^3 z^2 & xy^2 z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y} xy^2 z - \frac{\partial}{\partial z}(-2y^3 z^2)\right) - \hat{j}\left(\frac{\partial}{\partial x} xy^2 z - \frac{\partial}{\partial z} x^2 z\right) + \hat{k}\left(\frac{\partial}{\partial x}(-2y^3 z^2) - \frac{\partial}{\partial y} x^2 z\right)$$

$$= \hat{i}(2xyz + 4y^3 z) - \hat{j}(y^2 z - x^2) + \hat{k}(-0)$$

Now

$$(\nabla \times \bar{A})_{(1,-1,1)} = \hat{i}(-2 + 4(-1)^3(1)) - \hat{j}((-1)^2(1) - 1^2)$$

$$= \underline{-6\hat{i}}.$$

Q.2 Show that $\bar{A} = 3y^4 z^2\hat{i} + 4x^3 z^2\hat{j} - 3x^2 y^2\hat{k}$ is solenoidal.

Q.3 Determine the constant b such that $\bar{A} = (bx + 4y^2 z)\hat{i} + (x^3 \sin z - 3y)\hat{j} - (e^x + 4\cos x^2 y)\hat{k}$ is solenoidal. (Ans $b = 3$)

Q.4 Determine the constants a and b such that curl of $(2xy + 3yz)\hat{i} + (x^2 + axz - 4z^2)\hat{j} + (3xy + 2byz)\hat{k}$ is zero. (Ans $3, -4$)

Q.5 Show that $\bar{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational.