

**Title :** AC Voltage Regulator: AC Voltage Controller with R and RL Load (Module-5) Date: 23/03/2020 Name of Faculty: Hiren Jariwala Lecture No: 5<sup>th</sup> (02:00 pm to 03:00 pm) Source of information: Power Electronics: Converter, Applications and Design, N. Mohan, T. M. Undeland, W. M. Robbins, Wiley India Edition

Single phase full wave ac voltage controller circuit using two SCRs or a single triac is generally used in most of the ac control applications. The ac power flow to the load can be controlled in both the half cycles by varying the trigger angle ' $\alpha$ '. Hence the full wave ac voltage controller is also referred to as to a bi-directional controller.



Instead of using two SCR's in parallel, a Triac can be used for full wave ac voltage control.

The thyristor  $T_1$  is forward biased during the positive half cycle of the input supply voltage. The thyristor  $T_1$  is triggered at a delay angle of ' $\alpha$ ' ( $0 \le \alpha \le \pi$ ).

The load current flows through the ON thyristor  $T_1$  and through the load resistor RL in the downward direction during the conduction time of  $T_1$  from  $\omega t = \alpha$  to  $\pi$  radians.



At  $\omega t = \pi$ , when the input voltage falls to zero the thyristor current (which is flowing through the load resistor RL) falls to zero and hence T<sub>1</sub> naturally turns off. No current flows in the circuit during  $\omega t = \pi$  to ( $\pi$ + $\alpha$ ).

The thyristor  $T_2$  is forward biased during the negative cycle of input supply and when thyristor  $T_2$  is triggered at a delay angle ( $\pi$ + $\alpha$ ), the output voltage follows the negative half cycle of input from  $\omega t = (\pi + \alpha)$  to  $2\pi$ .

The time interval (spacing) between the gate trigger pulses of  $T_1$  and  $T_2$  is kept at  $\pi$  radians or 180<sup>0</sup>.

At  $\omega t = 2\pi$ , the input supply voltage falls to zero and hence the load current also falls to zero and thyristor T<sub>2</sub> turn off naturally.

Input supply voltage

 $v_s = V_m \sin \omega t = \sqrt{2} V_s \sin \omega t$ ;

Output voltage across the load resistor  $R_L$ ;

$$v_o = v_L = V_m \sin \omega t$$
;  
for  $\omega t = \alpha$  to  $\pi$  and  $\omega t = (\pi + \alpha)$  to  $2\pi$ 

**Output load current** 

$$i_{O} = \frac{v_{O}}{R_{L}} = \frac{V_{m} \sin \omega t}{R_{L}} = I_{m} \sin \omega t \quad ;$$
  
for  $\omega t = \alpha$  to  $\pi$  and  $\omega t = (\pi + \alpha)$  to  $2\pi$ 



#### **Expression for The RMS Value of Output Voltage**

The RMS value of output voltage (load voltage) can be found using the expression

$$V_{O(RMS)}^{2} = V_{L(RMS)}^{2} = \frac{1}{2\pi} \int_{0}^{2\pi} v_{L}^{2} d(\omega t)$$

$$V_{L(RMS)}^{2} = \frac{1}{\pi} \int_{0}^{\pi} V_{m}^{2} \sin^{2} \omega t.d\omega t$$
$$V_{L(RMS)}^{2} = \frac{1}{2\pi} \int_{0}^{2\pi} v_{L}^{2}.d(\omega t) ;$$
$$v_{L} = v_{0} = V_{m} \sin \omega t ; \text{ For } \omega t = \alpha \text{ to } \pi \text{ and } \omega t = (\pi + \alpha) \text{ to } 2\pi$$

Hence,

$$V_{L(RMS)}^{2} = \frac{1}{2\pi} \left[ \int_{\alpha}^{\pi} (V_{m} \sin \omega t)^{2} d(\omega t) + \int_{\pi+\alpha}^{2\pi} (V_{m} \sin \omega t)^{2} d(\omega t) \right]$$
$$= \frac{1}{2\pi} \left[ V_{m}^{2} \int_{\alpha}^{\pi} \sin^{2} \omega t d(\omega t) + V_{m}^{2} \int_{\pi+\alpha}^{2\pi} \sin^{2} \omega t d(\omega t) \right]$$

$$\begin{split} &= \frac{V_m^2}{2\pi} \left[ \int_{\alpha}^{\pi} \frac{1 - \cos 2\omega t}{2} d(\omega t) + \int_{\pi+\alpha}^{2\pi} \frac{1 - \cos 2\omega t}{2} d(\omega t) \right] \\ &= \frac{V_m^2}{2\pi \times 2} \left[ \int_{\alpha}^{\pi} d(\omega t) - \int_{\alpha}^{\pi} \cos 2\omega t . d(\omega t) + \int_{\pi+\alpha}^{2\pi} d(\omega t) - \int_{\pi+\alpha}^{2\pi} \cos 2\omega t . d(\omega t) \right] \\ &= \frac{V_m^2}{4\pi} \left[ (\omega t) \Big/_{\alpha}^{\pi} + (\omega t) \Big/_{\pi+\alpha}^{2\pi} - \left[ \frac{\sin 2\omega t}{2} \right]_{\alpha}^{\pi} - \left[ \frac{\sin 2\omega t}{2} \right]_{\pi+\alpha}^{2\pi} \right] \\ &= \frac{V_m^2}{4\pi} \left[ (\pi - \alpha) + (\pi - \alpha) - \frac{1}{2} (\sin 2\pi - \sin 2\alpha) - \frac{1}{2} (\sin 4\pi - \sin 2(\pi + \alpha)) \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) - \frac{1}{2} (0 - \sin 2\alpha) - \frac{1}{2} (0 - \sin 2(\pi + \alpha)) \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin 2(\pi + \alpha)}{2} \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin (2\pi + 2\alpha)}{2} \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin (2\pi + 2\alpha)}{2} \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin (2\pi + 2\alpha)}{2} \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin (2\pi + 2\alpha)}{2} \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin (2\pi + 2\alpha)}{2} \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin (2\pi + 2\alpha)}{2} \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin (2\pi + 2\alpha)}{2} \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin (2\pi + 2\alpha)}{2} \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin (2\pi + 2\alpha)}{2} \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin (2\pi + 2\alpha)}{2} \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin (2\pi + 2\alpha)}{2} \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin (2\pi + 2\alpha)}{2} \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin (2\pi + 2\alpha)}{2} \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin (2\pi + 2\alpha)}{2} \right] \\ &= \frac{V_m^2}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin (2\pi + 2\alpha)}{2} \right]$$

 $\sin 2\pi = 0 \& \cos 2\pi = 1$ 

Therefore,

 $V_{L(RMS)}^{2} = \frac{V_{m}^{2}}{4\pi} \left[ 2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin 2\alpha}{2} \right]$  $= \frac{V_{m}^{2}}{4\pi} \left[ 2(\pi - \alpha) + \sin 2\alpha \right]$  $V_{L(RMS)}^{2} = \frac{V_{m}^{2}}{4\pi} \left[ (2\pi - 2\alpha) + \sin 2\alpha \right]$ 

Taking the square root, we get

$$V_{L(RMS)} = \frac{V_m}{2\sqrt{\pi}} \sqrt{\left[\left(2\pi - 2\alpha\right) + \sin 2\alpha\right]}$$
$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}\sqrt{2\pi}} \sqrt{\left[\left(2\pi - 2\alpha\right) + \sin 2\alpha\right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{2\pi}} \Big[ (2\pi - 2\alpha) + \sin 2\alpha \Big]$$
$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{2\pi}} \Big[ 2 \Big\{ (\pi - \alpha) + \frac{\sin 2\alpha}{2} \Big\} \Big]$$
$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi}} \Big[ (\pi - \alpha) + \frac{\sin 2\alpha}{2} \Big]$$
$$V_{L(RMS)} = V_{i(RMS)} \sqrt{\frac{1}{\pi}} \Big[ (\pi - \alpha) + \frac{\sin 2\alpha}{2} \Big]$$
$$V_{L(RMS)} = V_s \sqrt{\frac{1}{\pi}} \Big[ (\pi - \alpha) + \frac{\sin 2\alpha}{2} \Big]$$

A single phase full wave ac voltage controller circuit (bidirectional controller) with an RL load using two thyristors  $T_1$  and  $T_2$  connected in parallel is shown in the figure below.

The thyristor  $T_1$  is forward biased during the positive half cycle of input supply. Let us assume that  $T_1$  is triggered at  $\omega t = \alpha$ , by applying a suitable gate trigger pulse to  $T_1$  during the positive half cycle of input supply. The output voltage across the load follows the input supply voltage when  $T_1$  is ON.



The load current  $i_0$  flows through the thyristor  $T_1$  and through the load in the downward direction.

Due to the inductance in the load, the load current  $i_0$  flowing through T<sub>1</sub> would not fall to zero at  $\omega t = \pi$ , when the input supply voltage starts to become negative.

The thyristor  $T_1$  will continue to conduct the load current until all the inductive energy stored in the load inductor L is completely utilized and the load current through  $T_1$  falls to zero at  $\omega t = \beta$ , where  $\beta$ is referred to as the Extinction angle, (the value of  $\omega t$ ) at which the load current falls to zero.

The extinction angle  $\beta$  is measured from the point of the beginning of the positive half cycle of input supply to the point where the load current falls to zero.

The thyristor  $T_1$  conducts from  $\omega t = \alpha \text{ to } \beta$ . The conduction angle of  $T_1$  is  $\delta = (\beta - \alpha)$ , which depends on the delay angle  $\alpha$  and the load impedance angle  $\phi$ .





#### **Expression for The RMS Value of Output Voltage**

When  $\alpha > \phi$ , the load current and load voltage waveforms become discontinuous

 $V_{O(RMS)} = \left[\frac{1}{\pi}\int_{\alpha}^{\beta} V_m^2 \sin^2 \omega t.d(\omega t)\right]^{\frac{1}{2}}$ 

Output  $v_o = V_m \sin \omega t$ , for  $\omega t = \alpha$  to  $\beta$ , when  $T_1$  is ON.

$$V_{O(RMS)} = \left[\frac{V_m^2}{\pi}\int_{\alpha}^{\beta} \frac{(1-\cos 2\omega t)}{2}d(\omega t)\right]^{\frac{1}{2}}$$
$$V_{O(RMS)} = \left[\frac{V_m^2}{2\pi}\left\{\int_{\alpha}^{\beta} d(\omega t) - \int_{\alpha}^{\beta} \cos 2\omega t.d(\omega t)\right\}\right]^{\frac{1}{2}}$$
$$V_{O(RMS)} = \left[\frac{V_m^2}{2\pi}\left\{(\omega t)\Big/_{\alpha}^{\beta} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\beta}\right\}\right]^{\frac{1}{2}}$$
$$V_{O(RMS)} = \left[\frac{V_m^2}{2\pi}\left\{(\beta - \alpha) - \frac{\sin 2\beta}{2} + \frac{\sin 2\alpha}{2}\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m \left[ \frac{1}{2\pi} \left\{ \left(\beta - \alpha\right) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2} \right\} \right]^{\frac{1}{2}}$$
$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \left[ \frac{1}{\pi} \left\{ \left(\beta - \alpha\right) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2} \right\} \right]^{\frac{1}{2}}$$

The RMS output voltage across the load can be varied by changing the trigger angle  $\alpha$ .

# Assignment

1. Explain single phase full controller with RL load with detailed analysis.

#### Hard copy submission date (03/4/2020).

For any kind of Query, do contact on following contact any time. Mr. Hiren Jariwala (9998838416)