Title : Routh Locus

Date:23/03/2020

Name of Faculty: Sagar Jani

Lecture No : 1

Source of information : "Process Systems Analysis and Control" By Donald R. Coughanowr and "Chemical Process Control An Introduction to Theory and Practice" By George Stephanopoulos

Routh Locus

- It is graphical technique
- Graph is obtained by plotting the roots of Characteristic equation in the complex plane by changing the value of Controller gain.
- Location of all roots is observed in the plot
- By plotting the graph of roots of Ch. Eq., we can obtained the response of an arbitrary forcing function

Procedure for Root Locus diagram

- Step 1: Derive the open loop transfer function and from the transfer function generate the Characteristic equation(Ch. Eq.) for the system is:

$$1 + G_{OL} = 0$$

- Step 2: By solving the Ch. Eq. we get the roots of equation
- Step 3: Now by changing the value of controller gain we get the different roots of Ch. Eq.
- Step 4:By plotting the graph of Controller gain v/s Roots of Ch. Eq. on complex plan will obtained the Root Locus diagram
- Step 5: From the Root locus diagram we can know the response of the system.

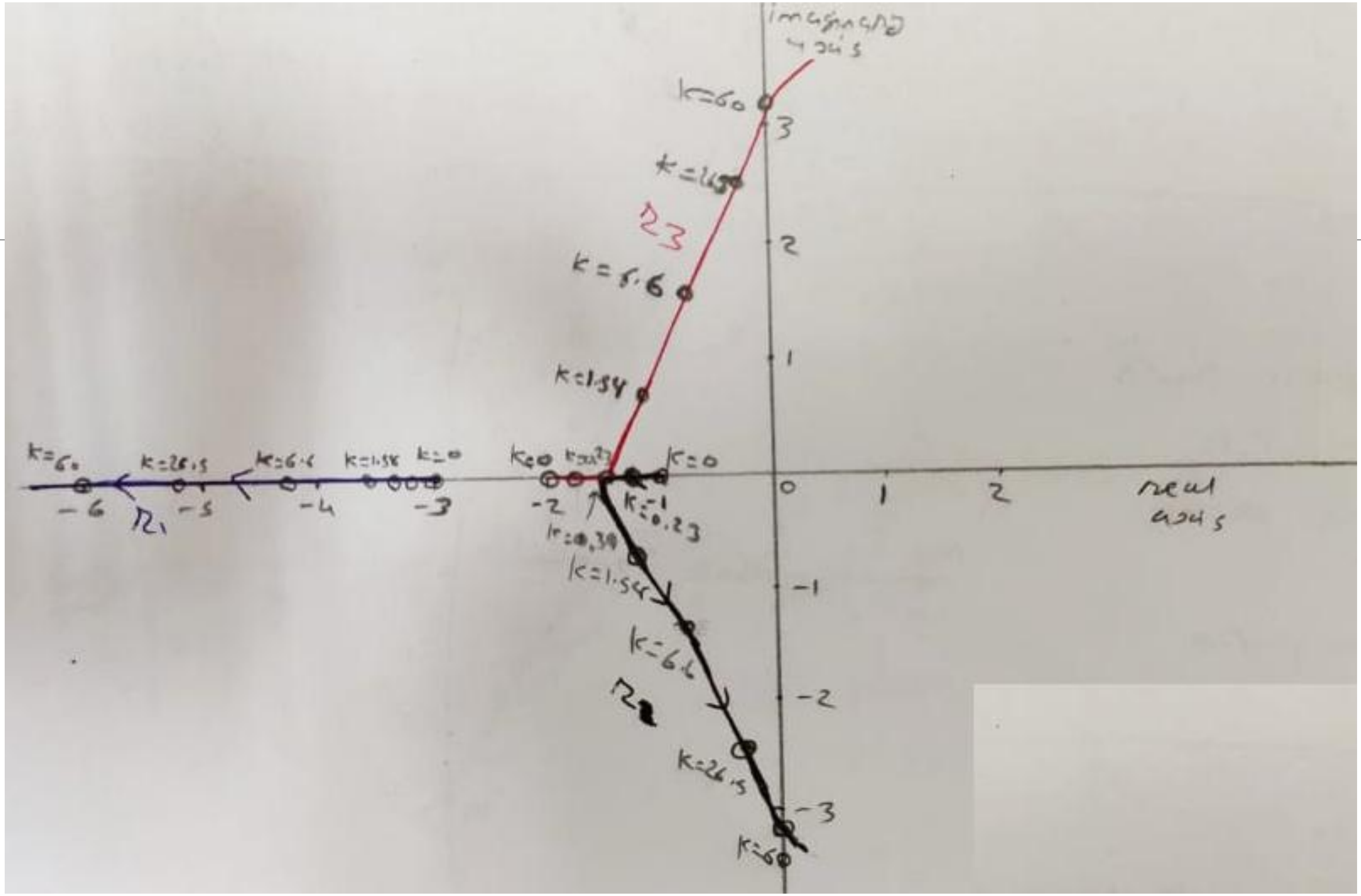
- For E.g., we have Ch. Eq.: $s^3 + 6s^2 + 11s + (6 + 6K_C) = 0$
- It's a third order polynomial Eq. so we get three set of roots

$$s^3 + 6s^2 + 11s + (6 + 6K) = 0$$

Where, $K = 6K_c$

- For the above Eq. by Changing the value of 'K', we get the different roots for this Eq. as follows:

	Root-1 (r_1)	Root-2 (r_2)	Root-3 (r_3)	
K=0	-3	-2	-1	
K=0.23	-3.1	-1.75	-1.15	
K=0.39	-3.16	-1.42	-1.42	$\Rightarrow K_2$
K=1.58	-3.45	$-1.28 - 0.75i$	$-1.28 + 0.75i$	
K=6.6	-4.11	$-0.95 - 1.5i$	$-0.95 + 1.5i$	
K=26.5	-5.1	$-0.45 - 2.5i$	$-0.45 + 2.5i$	
K=60	-6	$-3.32i$	$+3.32i$	$\Rightarrow K_3$
K=100	-6.72	$0.35 - 4i$	$0.35 + 4i$	



Root Locus Diagram

- The above table revels two critical value of K
  - K_2 at two roots become equals
  - K_3 at roots are pure imaginary

1. $K < K_2$: Response will be Non-oscillatory (because all roots are real)
2. $K_2 < K < K_3$: Response will be Oscillatory (because roots are complex)
3. $K > K_3$: Response will be Sinusiod (because roots have positive real parts)

Rules of Plotting Diagram

1. No. of branch/ Root Locii = No. of poles of open loop transfer function
2. The Root Locii originate from the poles of G_{OL} when Kc=0
3. The Root Locii terminate when Kc=∞
4. Closed loop system is stable irrespective of Kc value
5. Poles are away from real axis as the value of Kc increases, the response becomes more oscillatory.

6. The point at which two Root Locii emerging from adjusant poles on real axis, intersect & leave the real axis is determined by Solution of following Eq.

$$\sum_{i=1}^{m} \frac{1}{s - z_i} = \sum_{i=1}^{n} \frac{1}{s - p_j}$$

7. The gain at each value in root locus is obtained by solving the following Eq.

$$K \frac{|s - z_1||s - z_2| \ldots \ldots . |s - z_m|}{|s - p_1||s - p_2| \ldots \ldots . |s - p_n|} = 1$$

8. The point at which Root Locii crosses the imaginary axis can be found from the step No. 8 of Routh test Criteria