Mass Transport By Means of Diffusion and Convection

Previously, we have considered a case of diffusing component A through a non-diffusing component B. Schematically,

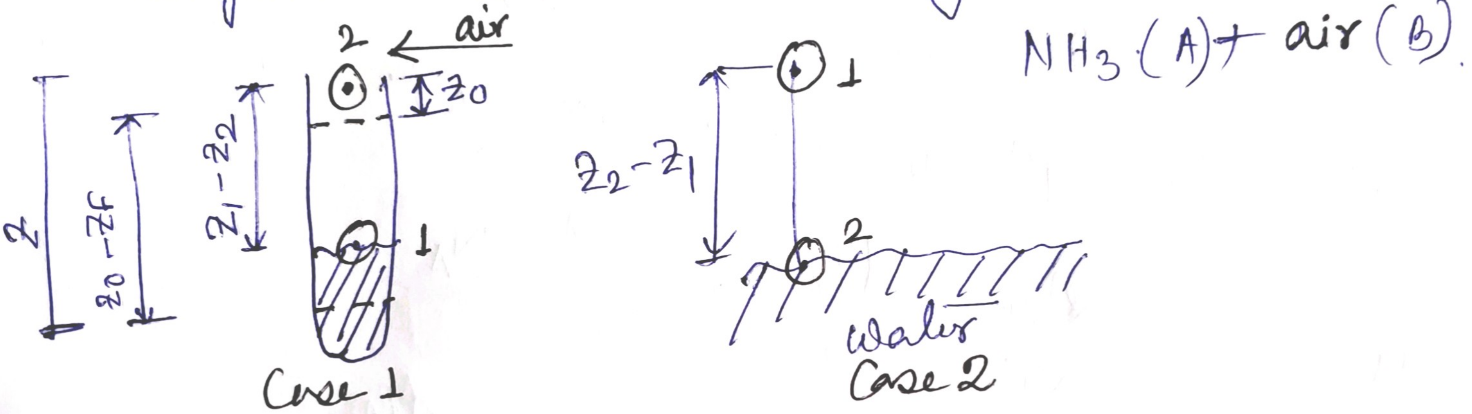


Fig. 1.

In fig. 1, case 1 presents water is getting evaporated from the air water interface. ~~from~~ Let's assume point 1 is at the air-water interface and point 2 far away from the surface. ~~In~~ In that case,

(1) $p_{A2} = 0$ as air is continuously flowing so the point does not contain any water molecule. Similarly, in case of case 2, NH_3 is dissolved into water and air has zero solubility in pure water. Thus, in both the cases, we can write general diffusive + convective

Mass transfer Eq^n given by Fick's law

$$N_A = -\mathcal{D}_{AB}\frac{dC_A}{dz} + \frac{C_A}{C}(N_A + \cancel{N_B}^{\to 0^*}) \quad \text{—— (i)}$$

* Since Component B has zero flux (i.e., non-diffusing component).

Consider, $C = \frac{P}{RT}$ and $p_A = x_A P$.

$$C_A = \frac{p_A}{RT} \quad \text{—— (ii)}$$

P: total Pressure
p_A: partial pressure of A
x_A = mole fraction of A.

Substituting Eq^n (ii) into (i) we get

$$N_A = -\frac{\mathcal{D}_{AB}}{RT}\frac{dp_A}{dz} + \frac{p_A}{P} N_A$$

$$\Rightarrow N_A\left(1 - \frac{p_A}{P}\right) = -\frac{\mathcal{D}_{AB}}{RT}\frac{dp_A}{dz}$$

$$\Rightarrow N_A\int_{z_1}^{z_2} dz = -\frac{\mathcal{D}_{AB}P}{RT}\int_{p_{A1}}^{p_{A2}}\frac{dp_A}{(P - p_A)}$$

$$\Rightarrow \boxed{N_A = \frac{\mathcal{D}_{AB}P}{RT(z_2 - z_1)} \cdot \ln\frac{P - p_{A2}}{P - p_{A1}}} \quad \text{—— (iii)}$$

Additionally, $P = p_{A1} + p_{B1} = p_{A2} + p_{B2}$

$$\Rightarrow p_{B1} = P - p_{A1}$$
$$p_{B2} = P - p_{A2}$$

We Introduce log-mean pressure difference (LMPD)

$$p_{BM} = \frac{p_{B2} - p_{B1}}{\ln(p_{B2}/p_{B1})}$$

The basic objective of Introducing LMPD is that the pressure or Concentration profile of A non-linear with the distance. Thus, we Introduce LMPD to Incorporate an logarithmic average of concentration or pressure as a correction factor.

So,

$$p_{BM} = \frac{(p_{A1} - p_{A2})}{\ln\left(\frac{P - p_{A2}}{P - p_{A1}}\right)}$$

Thus from Eq^n (iii) we get,

$$N_A = \frac{D_{AB} P}{RT (z_2 - z_1)} \cdot (p_{A1} - p_{A2}) \cdot \frac{1}{\frac{(p_{A1} - p_{A2})}{\ln\left(\frac{P - p_{A2}}{P - p_{A1}}\right)}}$$

$$\Rightarrow \boxed{N_A = \frac{D_{AB} P}{RT p_{BM} (z_2 - z_1)} \cdot (p_{A1} - p_{A2})} \quad \text{— (iv)}$$

Eq^n (iv) is valid for ~~cylindrical~~ xyz Coordinates, however, for spherical Coordinate system, more rigorous approach has to be taken. For example, Consider a case of Napthalene ball is kept at atmospheric Condition or a drop of water falls from evaporating from the surface. In such cases, we will derive an expression on spherical Co-ordinates in order to estimate N_A.

Consider the case of Napthalene ball evaporation;

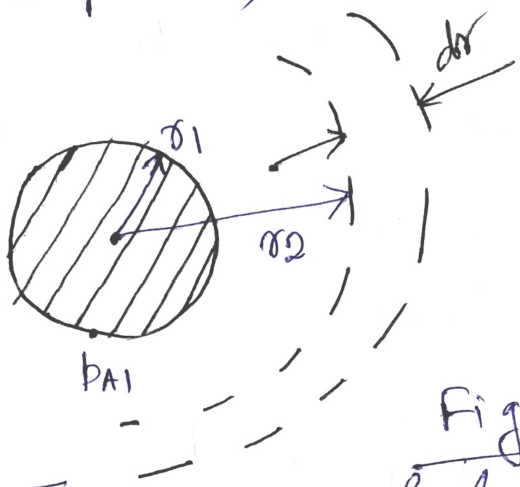


Fig. 2

So far we taken the surface area to be constant (i.e, per unit surface area) In case of a system with variable surface area we introduce, specific mass transport rate

$\bar{N}_A = N_{Ai} \cdot A_i$ [where A_i is the surface area of the diffusing component]

In other words, for the above case of Napthalene diffusion,

$$N_A = \frac{\bar{N}_A}{4\pi r^2} = -\frac{D_{AB}}{RT}\frac{dp_A}{(1-p_A/p)\,dr}$$

Since we are working in spherical Co-ordinates dz is Replaced by dz.

$$\Rightarrow \frac{\bar{N}_A}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r^2} = -\frac{D_{AB}}{RT}\int_{p_{A1}}^{p_{A2}}\frac{dp_A}{(1-p_A/p)}.$$

$$\Rightarrow \frac{\bar{N}_A}{4\pi}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) = \frac{D_{AB}P}{RT}\int_{p_{A1}}^{p_{A2}}\frac{dp_A}{(1-p_A/p}$$

Since, $r_2 >> r_1$ i.e, $1/r_2 \approx 0$

we get,

$$\Rightarrow \frac{\bar{N}_A}{4\pi r_1} = \frac{\mathcal{D}_{AB} P}{RT} \frac{p_{A1} - p_{A2}}{p_{BM}}$$

$$\Rightarrow \boxed{N_{A1} = \left(\frac{\mathcal{D}_{AB} P}{RT r_1}\right)\left(\frac{p_{A1} - p_{A2}}{p_{BM}}\right)}$$

Now, as Napthalene evaporates, the radius also decreases with time. If we want to consider a case where we would like to know at what point of time we will get a desired size / length of a particular item (i.e., could possibly be the liquid level after time $t = t_z$)

~~However~~, ~~considering the same Napthalene ball diffusing into air case, we get,~~

Considering the case described in Fig. 1. we would like to find out at what time liquid / water level would be z_f.

$$N_A \cdot \cancel{A} = \underbrace{\frac{\rho_A}{M_A}}_{\text{density}} \underbrace{\left(\frac{dz \cdot \cancel{A}}{dt}\right)}_{\text{volume term}}$$

$dt \to$ time

$M_A \to$ Mol. wt.

Again, we know, $N_A = C_A V_A$, Thus,

$$\Rightarrow \frac{\rho_A}{M_A} \cdot \frac{dz}{dt} = \frac{D_{AB} P}{z RT\, p_{BM}} (p_{A1} - p_{A2})$$

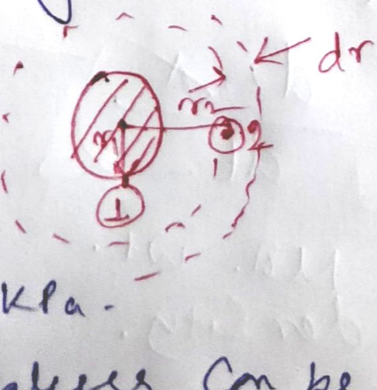
From Eq^n(iv)

$$\Rightarrow \frac{\rho_A}{M_A} \int_{z_0}^{z_f} z\, dz = \frac{D_{AB} P (p_{A1} - p_{A2})}{RT\, p_{BM}} \int_0^{t_f} dt$$

$$\Rightarrow \frac{\rho_A}{M_A} \left[z_f^2 - z_0^2 \right] = \frac{2 D_{AB} P}{RT\, p_{BM}} (p_{A1} - p_{A2})\, t_f$$

$$\Rightarrow \boxed{t_f = \left[\frac{RT\, \rho_A}{p_{BM} M_A} (z_f^2 - z_0^2) \right] \Big/ 2 D_{AB} P (p_{A1} - p_{A2})}$$

Problem 1. Find out the time required to completely evaporate a spherical Napthalene ball of the following specifications.



$T = 25.9\,°C$.

p_{A2} = you have to define

$r_1 = 2.00$ mm.

r_2 = you have to define.

$p_{A1} = 3.84$ kPa.

ρ_A, M_A = values can be find from literature

D_{AB} = literature

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