Reynolds Analogy

Reynolds Analogies

If dp*/dx*=0, Pr=Sc=1, the boundary layer equations for fluid flow, heat transfer and mass transfer become same form.

$$C_f \frac{\mathrm{Re}_L}{2} = Nu = Sh$$

$$St \equiv \frac{h}{\rho V c_p} = \frac{Nu}{\mathrm{Re}\,\mathrm{Pr}}$$

$$St_m \equiv \frac{h_m}{V} = \frac{Sh}{\mathrm{Re}\,Sc}$$

Pr = Sc =1

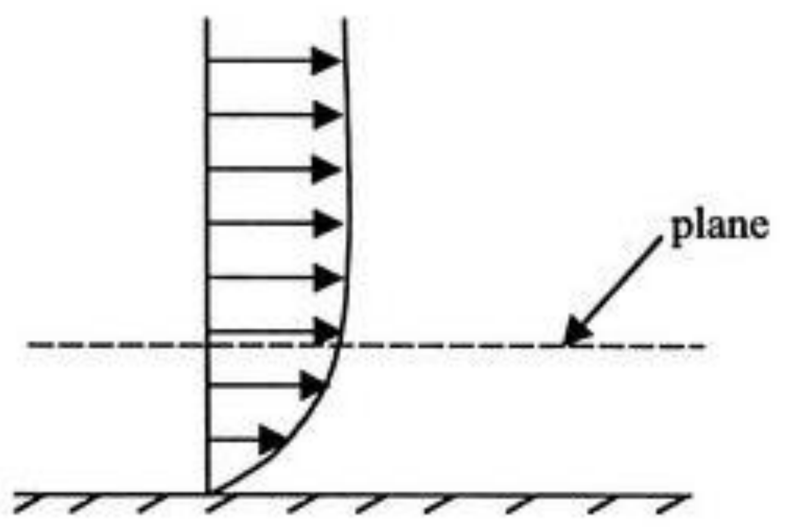
$$C_f / 2 = St = St_m$$

Description of analogy

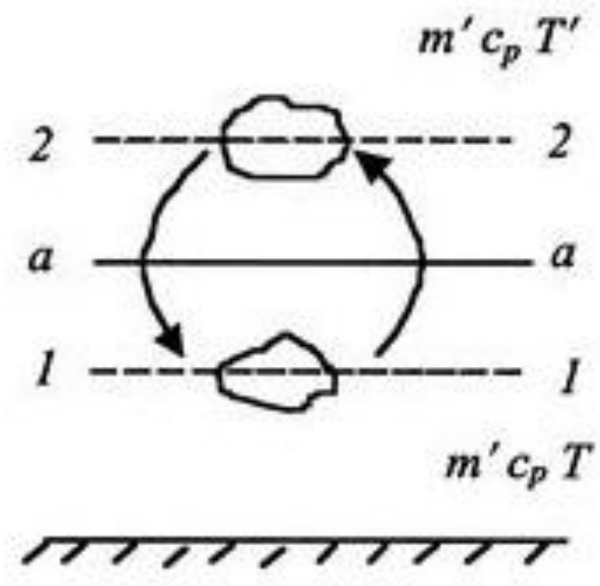
- We describe the physical mechanism for the heat transfer coefficient in a **turbulent boundary layer** because most aerospace vehicle applications have turbulent boundary layers.
- The treatment closely follows that in Eckert and Drake (1959).
- Very near the wall, the fluid motion is smooth and laminar, and molecular conduction and shear are important.
- The shear stress, at a plane is given by (where is the dynamic viscosity), and the heat flux
- The latter is the same expression that was used for a solid.

Description of analogy

- The latter is the same expression that was used for a solid.
- The boundary layer is a region in which the velocity is lower than the free stream as shown in Figure
- In a turbulent boundary layer, the dominant mechanisms of shear stress and heat transfer change in nature as one moves away from the wall.



Velocity profile near a surface

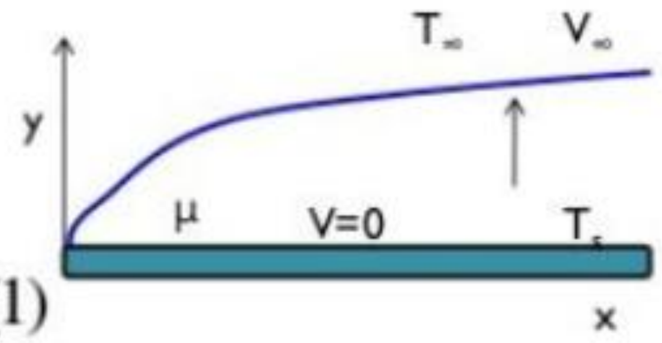


Momentum and energy exchanges in turbulent flow

Reynold's Analogy

Reynold's Analogy is the relationship between C_f & h (heat transfer by convection) between plate surface and fluid for Laminar Flow over Flat Plate

As per Newton's Law of Viscosity, Shear Stress in Laminar Flow in the normal direction to the Plate is given as:



$$\tau_s = \mu \frac{dV}{dy} \Rightarrow dy = \frac{\mu dV}{\tau_s} \text{........}(1)$$

Heat Flow along y direction is given by Fourier's Law

$$Q = -KA\frac{dT}{dy} \text{........................}(2)$$

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Reynold's Analogy

$$We\ know\ that\ \Pr = \frac{\mu C_p}{K}$$

$$Assuming\ \Pr \approx 1; we\ have\ K = \mu C_p \ ..(3)$$

$$On\ substitution\ in\ Eqn(2)\ from\ (1)\ \&\ (3)$$

$$Q = -\mu C_p A \frac{dT}{dy}$$

$$Q = -\mu C_p A \frac{dT}{\mu dV} \tau_s = -C_p A \frac{dT}{dV} \tau_s \ ......(4)$$

Reynold's Analogy

BC 1) For $V=0$ at plate surface, $T=T_s$

BC 2): For $V=V_\infty$ on outer edge of BL; $T=T_\infty$

Separating Variables and Integrating,
we have:

$$\frac{Q}{C_p.A.\tau_s}\int_0^{V\infty} dV = -\int_{Ts}^{T\infty} dT \Rightarrow \frac{Q}{C_p.A.\tau_s}.V_\infty = (T_s - T_\infty)$$

Reynold's Analogy

$$\Rightarrow \frac{Q}{A(T_s - T_\infty)} = \tau_s \frac{C_p}{V_\infty} \Rightarrow h = \tau_s \frac{C_p}{V_\infty}$$

Skin Friction is defined in Drag Force as :

$$F_D = C_f . \frac{1}{2} \rho A V^2$$

$$Hence \quad \tau_s = \frac{F_D}{A} = C_f . \frac{1}{2} \rho V^2$$

Reynold's Analogy (Pr=1)

$$Substituting\ in\ equation \Rightarrow h = \tau_s \cdot \frac{C_p}{V_\infty}$$

$$h = C_f \cdot \frac{1}{2} \rho V_\infty^{\ 2} \cdot \frac{C_p}{V_\infty}$$

$$\frac{C_f}{2} = \frac{h}{\rho C_p V_\infty} = St \quad REYNOLD's\ ANALOGY$$

Chilton-Colburn Analogies

$$\frac{C_f}{2} = St\,\Pr^{2/3} \equiv j_H$$ **0.6 < Pr < 60**

$$\frac{C_f}{2} = St_m Sc^{2/3} \equiv j_m$$ **0.6 < Sc < 3000**

For Laminar flow, need dP*/dx* =0;

For turbulent flow, doesn't need dp*/dx*=0

Chilton & Colburn Analogy

Reynold's Analogy assumes Pr=1; hence when Pr≠1, poor results are obtained. This analogy was modified Chilton & Colburn

We know that : $$Nu = 0.664\,\mathrm{Re}^{1/2}.\mathrm{Pr}^{1/3}$$

Dividing both sides by $\mathrm{Re}\,\mathrm{Pr}^{1/3}$ *; We have*

$$\frac{Nu}{\mathrm{Re}.\mathrm{Pr}^{1/3}} = \frac{0.664}{\mathrm{Re}^{1/2}} = \frac{1}{2}.\frac{1.328}{\sqrt{\mathrm{Re}}} = \frac{C_f}{2}$$

Chilton & Colburn Analogy

$$\frac{C_f}{2} = \frac{Nu}{Re.Pr^{1/3}} = St.Pr^{2/3}$$

CHILTON & COLBURN ANALOGY

$(Holds\ good\ for\ \mathrm{Pr}\ from\ 0.5\ to\ 50)$

$(Put\ \mathrm{Pr} = 1 \Rightarrow \mathrm{Re}ynold's\ Ana\mathrm{log}y)$

Questions:

1. What are the other analogies of transport phenomena. Explain in brief.

2. What is the importance of analogies from chemical engineering point of view.

3. Explain in details validity of Reynolds number and its significance.

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