SUBJECT: *CVNM*

TOPIC: *NEWTON'S FORWARD,BACKWARD DIFFERENCE INTERPOLATION*

NEWTON’S FORWARD DIFFERENCE INTERPOLATION FORMULA

$$y = f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots + \frac{p(p-1)(p-2)\ldots(p-(n-1))}{n!}\Delta^n y_0$$

Where $-1<p<1$

FIRST FORWARD DIFFERENCES

- The $y_1 - y_0$, $y_2 - y_1$, $y_n - y_{n-1}$.differences are called the first forward differences of the function.

- $y = f(x)$ and we denote these difference by

  Δy_0 , Δy_1 , Δy_2..........., Δy_n respectively, where Δ is called the descending or forward difference operator.

- In general, the first forward differences is defined by

$\Delta y_x = y_{x+1} - y_x$.

where Δ is called first forward difference operator.

SECOND FORWARD DIFFRENCE OPERATOR

- The differences of first forward differences are called second forward differences.

$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$.

$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$.

$\Delta^2 y_{n-1} = \Delta y_n - \Delta y_{n-1}$.

$\Delta^2 y_0$, $\Delta^2 y_1, \ldots\ldots\ldots\ldots, \Delta^2 y_{n-1}$ are called second forward differences.

- where Δ^2 is called second forward difference order.

TABLE

Argument x	Entry y = f(x)	First Differences Δ	Second Differences Δ^2	Third Differences Δ^3	Fourth Differences Δ^4
x_0	y_0				
		Δy_0			
x_1	y_1				
			$\Delta^2 y_0$		
		Δy_1			
x_2	y_2			$\Delta^3 y_0$	
			$\Delta^2 y_1$		
		Δy_2			$\Delta^4 y_0$
x_3	y_3			$\Delta^3 y_1$	
			$\Delta^2 y_2$		
		Δy_3			
x_4	y_4				

Illustration 1 : Find the value of $sin52°$ from the following :

$\theta°$	45°	50°	55°	60°
$sin\theta$	0.7071	0.7660	0.8192	0.8660

Solution : The table of difference is

$\theta°$	$sin\theta$	Δ	Δ^2	Δ^3
45°	0.7071			
		0.0589		
50°	0.7660		-0.0057	
		0.0532		-0.0007
55°	0.8192		-0.0064	
		0.0468		
60°	0.8660			

∴ Applying Newton's forward difference interpolation formula.

$$y_n(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

Here $y_n(x) = sin52°$

$y_0 = 0.7071,\quad \Delta y_0 = 0.0589,\quad \Delta^2 y_0 = -0.0057,\quad \Delta^3 y_0 = -0.0007$

$$p = \frac{x - x_0}{h} = \frac{52 - 45}{5} = \frac{7}{5} = 1.4$$

$$= 0.7071 + (1.4)(0.0589) + \frac{1.4(0.4)}{2}(-0.0057) + \frac{1.4(0.4)(-0.6)}{6}(-0.007)$$

$$= 0.7071 + 0.08246 - 0.0016 + 0.00004$$

$\therefore \boldsymbol{sin52° = 0.7880}$

Illustration 2: Using Newton's forward formula, find the value of f(218) if,

X	100	150	200	250	300	350	400
F(x)	10.63	13.03	15.04	16.81	18.42	19.90	21.27

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Solution : The table difference is

X	f(x)	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
100	10.63						
		2.4					
150	13.03		-0.39				
		2.01		0.15			
200	15.04		-0.24		-0.07		
		1.77		0.08		0.02	
250	16.81		-0.16		-0.05		0.02
		1.61		0.03		0.04	
300	18.42		-0.13		-0.01		
		1.48		0.02			
350	19.90		-0.11				
		1.37					
400	21.27						

Solution : We have,

$$y_n(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

Here $y_n(x) = f(218)$, $\quad p = \frac{x - x_0}{h} = \frac{218 - 100}{50} = 2.36$

$$\therefore f(218) = 10.63 + \frac{(2.36)(1.36)}{2}(-0.39) + \frac{(2.36)(1.36)(0.36)}{6}(0.15)$$

$$+ \frac{(2.36)(1.36)(0.36)(-0.64)}{24}(-0.07) + \frac{(2.36)(1.36)(0.36)(-0.64)(-1.64)}{120}(0.02)$$

$$+ \frac{(2.36)(1.36)(0.36)(-0.64)(-1.64)(-2.64)}{720}(0.02)$$

$$= 10.63 - 0.6259 + 0.0289 + 0.0022 + 0.0002 - 0.0001$$

$$\therefore \boldsymbol{f(218) = 10.0353}$$

Newton’s backward interpolation

FIRST BACKWARD DIFFRENCES

- The y_1- y_0 ,y_2 - y_1 ,y_n - y_{n-1}.differences are called the first forward differences of the function.

- $y = f(x)$ and we denote these difference by

∇y_1 , ∇y_2 , ∇y_3..........,∇y_nrespectively, where $\nabla$is called the descending or forward difference operator.

- In general, the first forward differences is defined by

$\nabla y_n = y_n - y_{n-1}$.

where is called first backward difference operator.

SECOND BACKWARD DIFFRENCE OPERATOR

- The differences of first forward differences are called second backward differences.

  $\nabla^2 y_1 = \Delta y_1 - \Delta y_0$.

  $\nabla^2 y_2 = \Delta y_2 - \Delta y_1$.

  $\nabla^2 y_n = \Delta y_n - \Delta y_{n-1}$.

  $\nabla^2 y_1$, $\Delta^2 y_2, \ldots\ldots\ldots\ldots, \Delta^2 y_n$ are called second forward differences.

- where ∇ is called second backward difference operator.

TABLE

Argument x	Entry y = f(x)	First Differences ∇	Second Differences ∇^2	Third Differences ∇^3	Fourth Differences ∇^4
x_0	y_0				
		∇y_0			
x_1	y_1		$\nabla^2 y_0$		
		∇y_1		$\nabla^3 y_0$	
x_2	y_2		$\nabla^2 y_1$		$\nabla^4 y_0$
		∇y_2		$\nabla^3 y_1$	
x_3	y_3		$\nabla^2 y_2$		
		∇y_3			
x_4	y_4				

Applying Newton's Backward Difference interpolation Formula.

$$y_n(x) = y_0 + p\nabla y_n + \frac{p(p-1)}{2!}\nabla^2 y_n + \frac{p(p-1)(p-2)}{3!}\nabla^3 y_n + \frac{p(p-1)(p-2)(p-3)}{4!}\nabla^4 y_n$$

Here:- $y_n(x) = yn(300)$

$$\therefore p = \frac{x-xn}{h} = \frac{300-250}{50} = 1$$

$$\therefore y_n(x) = 1032 + 126 + \frac{1(1+1)}{2!}25 + \frac{1(1+1)(1+2)}{3!}5 + \frac{1(1+1)(1+2)(1+3)}{4!}(-40)$$

$$= 1032 + 126 + 25 + 5 - 4$$

$$y_n(300) = 1148$$